THE MASTER THEOREM FOR DIVIDE-AND-CONQUER RECURRENCES COMPSCI 320

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There are many versions of this in the literature. What follows is not the most general, but it is enough for our purposes.

Here we assume that f is a function on the natural numbers with $f(n) \geq 0$, n_0 a natural number, $a > 0$, b a natural number with $b \geq 2$, $c > 0$.

Theorem. *Suppose that T is an increasing function on $\mathbb{N}$ that satisfies $T(n_0) = c$ and $T(n) = aT(n/b) + f(n)$ whenever $n/n_0 = b^k$ for some integer $k \geq 0$.*

Define $e = \log_b a$. Then if $f(n) \in \Theta(n^d (\lg n)^q)$ for $d, q \geq 0$, we have

$$T(n) \in \begin{cases} \Theta(n^e) & \text{if } d < e; \\ \Theta(f(n) \lg n) & \text{if } d = e; \\ \Theta(f(n)) & \text{if } d > e. \end{cases}$$

Proof. Ask me, find it in the literature (check my handouts directory), or work it out yourself. □

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