

1. Exercise 8.1 from Kleinberg-Tardos.
2. Exercise 8.2 from Kleinberg-Tardos.
3. Exercise 8.6 from Kleinberg-Tardos.
4. Exercise 8.30 from Kleinberg-Tardos.
5. Suppose we have a biased coin with probability p of landing heads. We don't know the value of p , but we do know that $0 < p < 1$. How can we use this to generate uniformly random bits (in other words, to simulate a sequence of fair coin tosses)? What is the expected time for your algorithm to generate a single random bit? Hint: consider how to generate two events of equal probability by flipping your coin.
6. Suppose that we have an unbiased p -correct Monte Carlo algorithm for a problem for which there are $l > 2$ possible answers, only one of which is correct. One way of trying to amplify the stochastic advantage is to repeat the algorithm k times and choose the most frequently returned answer. Show that for $k = 3$ this can give an algorithm that is less likely to return the correct answer than the original. Hint: it suffices to find values of p and l for which this is true.
7. Suppose that you have biased Monte Carlo algorithms A and B for the same decision problem. Algorithm A is p -correct, and its answer is guaranteed when it returns TRUE; algorithm B is q -correct, and its answer is guaranteed when it returns FALSE. Show how to combine A and B into a Las Vegas algorithm to solve the same problem. If r denotes the success probability of your algorithm, what is the best value of r you can get?