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Abstracts: Constructivity, Complexity, and Fuzziness (CCF '99)

D.S. Bridges, C.S. Calude and L.S. Dediu (Editors)

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Constructivity, Complexity, and Fuzziness (CCF '99)

University "Dunărea de Jos"
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Abstracts

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Introduction

These are the abstracts of talks to be given at the Workshop *CCF '99 (Constructivity, Complexity, and Fuzziness)* to be held at the University "Dunărea de Jos", Galaţi, Romania, on 26–28 August 1999. The workshop was organised by the University "Dunărea de Jos", Galaţi, Romania, the Centre for Discrete Mathematics and Theoretical Computer Science, University of Auckland, New Zealand and the Department of Mathematics and Statistics, University of Canterbury, Christchurch, New Zealand.

In most cases the abstract refers to a single lecture on an aspect of one of the three subjects in the title of the workshop; but Bridges will give two lectures, introducing modern developments in constructive mathematics. Invited speakers are D. S. Bridges, T. Buhăescu, C. .S. Calude, H. Ishihara, P. Odifreddi and L. Staiger.

The Conference Committee for the workshop consisted of the following people: Douglas S. Bridges, Canterbury, New Zealand; Cristian S. Calude, Auckland, New Zealand; Luminiţa Simona Dediu, Canterbury, New Zealand, and Galaţi, România; Mihaela Baroni, Galaţi, România. The Program Committee included the following people: Douglas S. Bridges; Toader Buhaescu, Galaţi, România; Cristian S. Calude; Luminiţa Simona Dediu; Peter Schuster, München, Germany; Ludwig Staiger, Halle, Germany; Doru Stefanescu, Bucharest, România; Petru Vâţă, Galaţi, România.

The actual organisation of the workshop was carried out by staff at the University of Galaţi, led by Professor Toader Buhaescu. Luminiţa Simona Dediu did much of the spade–work in the final weeks of preparation. We are most grateful to all those who put in so much effort to ensure that the meeting was a success, both professionally and socially. Special thanks are due to the Dean of the Faculty of Sciences of the University of Galaţi, Associate Professor Petru Vâţă.

For more details about the Workshop *CCF '99* see

`http://www.informatik.uni-halle.de/ staiger/galati.html`.

D.S. Bridges
C.S. Calude
L.S. Dediu
Christchurch, Auckland, Galaţi
July 1999

Abstracts

Semi–Qualitative Encoding of Manifestations at Faults in Conductive Flow Systems

Viorel Ariton
The University "Dunărea de Jos", Galaţi,
Romania
E-mail: vio@cs.ugal.ro

The fault diagnosis problem of real systems involves approximate reasoning because of the imprecise data and the incomplete knowledge of the human expert on the system faulty behavior. Furthermore, the human diagnostician knowledge refers to qualitative relations between linguistic variables on the observations made during the system running. The paper is a study on the fuzzy encoding of manifestations at fault, as deviations of the power variables (i.e. pressure like and flow-rate like variables) from the normal values, in conductive flow systems. The semi-qualitative encoding concerns the possibility sub-domains of a power variable related to the target system normal and abnormal behavior. In the presented approach, at knowledge acquisition phase, the expert must assert only the normal sub-domain, and only for the power variables directly related to the process ends (goals). Meaningful sub-domains for other variables - about which the human expert has few or no information on the propagated effects, may be deduced based on specific qualitative relations between power variables in the systems that involve flow conduction. The conductive flow system model is meant in terms of bond graphs, and the power variables are fuzzy variables that enter specific balance equations, stated by the conduction laws and the power restrictions that apply in the real system's running. The paper presents a new fuzzy arithmetic approach, appropriate to fuzzy addition of the intensive/extensive power variables, as negative correlated variables. With the proposed method one may obtain fuzzy subsets for the summed variables, in the bond graph junctions of the target conductive flow system, then obtain the fuzzy partition of the variables with no prior knowledge on the secondary effects at fault.

Constructive Mathematics—A Modern Perspective

Douglas S. Bridges
University of Canterbury
Christchurch, New Zealand
E-mail: d.bridges@math.canterbury.ac.nz

Although the origins of modern constructive mathematics lie in the philosophical–polemical work of L.E.J. Brouwer, most of the development of the subject has taken place in the past 42 years, following the appearance of Errett Bishop's monograph *Foundations of Constructive Analysis*, in which, for the first time, it was clearly shown that a systematic development of twentieth–century analysis could be carried through constructively. Since then, constructive methods have been applied successfully across a wide range of mathematics, covering algebra, analysis, topology, and even mathematical economics.

In these lectures I will present constructive mathematics, in a philosophy–independent fashion, as a natural way of approaching questions of computability in mathematics by changing the logic, rather than by working with a restrictive notion of algorithm. A tentative plan for the lectures is the following.

Lecture 1. Origins of constructivism in mathematics: Brouwer, Markov, recursive mathematics, and Bishop. The multiplicity of interpretations of constructive proofs. Intuitionistic logic and ZF set theory. Constructive axioms for the real line $\mathbf{R}$. The completeness of $\mathbf{R}$ and its peculiarly constructive applications. Lebesgue measure.

Lecture 2. The constructive theory of metric, normed, and Hilbert spaces. Recent developments in constructive operator and operator algebra theory (adjoints, numerical ranges, ultraweakly continuous linear functionals, ...).

It is unlikely that I will be able to cover all the topics mentioned above, but it should be possible to cover sufficient to give mathematicians, from senior undergraduate level up, a feel for the style and scope of an alternative approach to computable mathematics.

Uninorm Aggregation Operators and Implication Operators in the Intuitionistic Fuzzy Sets Class

Toader T. Buhăescu
The University "Dunărea de Jos", Galaţi
Romania
E-mail: tbuhaescu@math.ugal.ro

It is known from the fuzzy literature that the concept of implication operator is vital for theoretical development as well as for the practical applications. The present paper deals with the implication opearators in the theory of intuitionistic fuzzy sets, that is, in the intuitionistic fuzzy logic.

The collection of intuitionistic fuzzy sets in the universe E is denoted by $IFS(E)$ and defined as follows

$$IFS(E) = \{A = \langle A^+, A^- \rangle : A^+, A^- : E \to [0,1] \,\text{and}\, \forall x \in E(A^+(x) + A^-(x) \le 1)\}.$$

The set operations in $IFS(E)$ induce an algebraical structure which is isomorphic to $\{L, \vee, \wedge, c\}$, where

$$\begin{aligned} L &= \{a = \langle a^+, a^- \rangle : a^+, a^- \in [0,1] \,\text{and}\, a^+ + a^- \le 1)\} \\ a \vee b &= \langle a^+ \vee b^+, a^- \wedge b^- \rangle \\ a \wedge b &= \langle a^+ \wedge b^+, a^- \vee b^- \rangle \\ {}^c a &= \langle a^+, a^- \rangle \\ x \vee y &= max(x, y) \\ x \wedge y &= min(x, y). \end{aligned}$$

Yager and Ribalov ([4]) noticed that t–norms and t–conorms have the same properties, except for the unit element (t–norms have 1, t–conorms have 0 as a unit element). They took a single element e to be the unit for both operations.

An implication operator in the intuitionistic fuzzy logic is a mapping $I : L \times L \to L$ of the form $I(a, b) = \langle I^+(a^-, b^+), I^-(a^+, b^-) \rangle$, with $I^+, I^- : [0,1] \times [0,1] \to [0,1]$. The mappings I^+, I^- play the role of the implication operators in the fuzzy logic. We study the properties of the intuitionistic fuzzy implication operators in general and in particular those regarding the axioms of Smets and Magrez. Also we propose a generalized modus ponens, using an uninorm aggregator and some intuitionistic fuzzy operators.

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Incompleteness and Constructivity

C.S. Calude
The University of Auckland
Auckland, New Zealand
E-mail: `cristian@cs.auckland.ac.nz`

A constructive analysis of the incompleteness phenomenon, based on classical results by Gödel, Turing and Chaitin, and (very) recent refinements proposed by Solovay and Raatikainen, will be presented. In particular, a positive answer to the question "does incompleteness concern (constructive) mathematics?" will be argued. Finally, a few open problems will be reviewed.

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Generalizing Fuzziness–An Algebraic Challenge

Rodica Ceterchi
The University of Bucarest
Romania
E-mail: rc@funinf.math.unibuc.ro

The algebraic counterpart of fuzzy set theory is the concept of Many-Valued algebra (MV algebra), which plays, with respect to many-valued logic. the same role as boolean algebra plays with respect to classical two-valued logic.

Recently, a non-commutative generalization of MV algebras was proposed by G. Georgescu and A. Iorgulescu, under the name of pseudo–MV algebras. An equivalent concept, that of pseudo-Wajsberg algebra was introduced and studied by the author.

The purpose of our paper is to present this concept, together with other, even weaker generalizations. All these concepts present a challenge as to their interpretation in terms of “fuzziness”.

Compromise and Fuzzy Object in Schedule’s Problem

Liliana Cucu
The University “Dunărea de Jos”, Galaţi
Romania
E-mail: lcucu@math.ugal.ro

Chromatic number of a graph gives solution for the problem of conflicts when the conflicts are seen as edges and the adjacent vertex as contradictory elements. When there are too many constraints, we are no longer looking for the optimal solution, but for an admissible one. This leads us to considering the notion of compromise. To model the compromise we use values from 0 to 1 according to the intensity of conflicts. We add a program in Borland Delphi to illustrate the theory. We conclude with an application in problem of schedule that permits to compromise.

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Embedding a Linear Subset of $\mathcal{B}(H)$ in the Dual of its Predual

Luminiţa Dediu
The University of Canterbury, Christchurch
New Zealand
and
The University "Dunărea de Jos", Galaţi
Romania E-mail: `lde15@student.canterbury.ac.nz; ldediu@math.ugal.ro`

In this paper we continue the study of spaces of operators on a Hilbert space within constructive mathematics, as part of a programme for the systematic constructive development of the theory of operator algebras (see [5], [6]). The embedding of a linear set of bounded operators on a separable Hilbert space as a dense subset of the dual of its predual is explored constructively.

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A Characterization of the Free-Extendible Prefix Free Sets and its Use in Extending the Kraft-Chaitin Theorem

Cristian Grozea
The University of Bucarest
Romania
E-mail: chrisg@phobos.cs.unibuc.ro

First, the dual set of a finite prefix free set is defined. Using this, a theorem is then proved, that describes the equivalent conditions for a finite prefix free set to be indefinitely extendible. Finally, there is a discussion on the influence of the alphabet size over the indefinite extensibility property.

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Feasibly Constructive Analysis

Hajime Ishihara
School of Information Science
Japan Advanced Institute of Science and Technology
Tatsunokuchi, Japan
E-mail: ishihara@jaist.ac.jp

In the constructive theory of real numbers developed, for example in [4, Chapter 5], we assume that a universe $\mathcal{U}$ of sequences of natural numbers satisfies certain closure conditions; a very weak axiom of choice QF-AC$_{00}$ expressing the fact that $\mathcal{U}$ is closed under *recursive in* is assumed in [4, Chapter 5].

On the other hand, various classes of functions on (sequences of) natural numbers have been defined as function algebras [1]; a *function algebra* is the smallest class of functions containing certain initial functions and closed under certain operations (especially composition and recursion scheme). For example, A. Cobham [2] characterized the polynomial time computable functions as the smallest class closed under *bounded recursion on notation*; see [3] for other characterizations of the polytime functions.

We give some elementary results and problems on the constructive theory of real numbers and analysis with a universe $\mathcal{U}$ which is closed under a recursion scheme characterizing the polytime functions.

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Computable p–adic Numbers

George Kapoulas
Economics Department
University of Crete
Greece
E-mail: gkapou@math.ntua.gr

In the present the notion of a *computable p–adic number* is introduced. The definition is analogous to the definition of the computable real number. We consider the topological completions of the field of the rational numbers w.r.t the p–adic metric and impose consructivity restrictions on the notion of Cauchy sequence in order to obtain the computable, primitive recursive and polynomially time computable p–adic numbers. We study the properties of the field of the computable p–adic numbers and abstract the properties of the natural numbers that represent the computable p–adic numbers and obtain characterizations of such sets of natural numbers representing the computable p–adic numbers.

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The Recursive Universe

Piergiorgio Odifreddi
Turin University
Turin, Italy
E-mail: piergior@di.unito.it

The universe of recursive sets is a microworld of the universe of all sets. In this talk I will give an overview of subrecursive classes, from very small time and space bounds to very fast-growing functions. The study of recursive sets is worth pursuing not only for its own sake, due to the importance of such sets, but also because it shows the interconnections of a number of different branches of logic, from complexity theory to proof theory.

A few words will be spent on the problem of classifying the recursive sets completely, and arguments will be given to show that a complete and natural classification is impossible to obtain.

2D Orthogonal Grid Generation with Elliptic Partial Differential Equations

Florin Popescu
The University "Dunărea de Jos", Galaţi
Romania
E-mail: fipopes@math.ugal.ro

A numerical method and a MATLAB code for two-dimesional orthogonal grid generation is presented. The code generates body fitted coordinates by solving a pair of elliptic partial differential equations for the coordinates x and y, subject to boundary conditions which the user dictates by specifying the geometry and number of nodes. The solution method is succesive over-relaxation (SOR). An investigation of the effects of P and Q terms on interior grid points distribution is performed.

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Ideals Versus Coideals in Constructive Mathematics

Peter M. Schuster
Mathematisches Institut der Universität München
München, Germany
E-mail: pschust@rz.mathematik.uni-muenchen.de

Based on the various concepts of a complement that appear in constructive mathematics, we undertake an investigation of the interplay between ideals and coideals in a commutative ring with inequality. Whereas ideals appear naturally and can easily be presented by means of generators, coideals seem to constitute the more appropriate concept from some constructive points of view. Our work is intended to clarify the given situation, also with respect to the current practice of computational algebra. Particular attention is paid to chain conditions, to minimality and maximality properties, to the various constructive notions of prime (co)ideals, and to the relationship of (co)ideals with the corresponding quotient rings.

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Complexity and Dimension

Ludwig Staiger
Martin-Luther-Universität Halle-Wittenberg
Institut für Informatik
Kurt-Mothes-Str.1
D–06120 Halle, Germany
E-mail: `staiger@cantor.informatik.uni-halle.de`

The concept of Kolmogorov or program size complexity measures the information content of a (finite) string as the size of the shortest program that computes the string, that is, the complexity of a string is the amount of information necessary to print the string.

For infinite strings the growth of the Kolmogorov complexity of the finite prefixes measures the amount of information which must provided in order to specify a particular symbol of this string. This shows that Kolmogorov complexity, as a theory, is a counterpart to the statistical information theory. Average information is known as entropy. Consequently, one expects that there are relationships between the theory of Kolmogorov complexity, which could be called Algorithmic information theory and classical information theory.

Starting from a well-known connection between the Kolmogorov complexity of finite words and a combinatorial kind of entropy, also known as Shannon capacity, we look for closer relationships between Algorithmic information theory and its classical counterpart. As a matter of fact, when looking for combinatorial rather than for probabilistic counterparts of entropy, it turns out that these can be found in geometric measure theory or, as it is now better known as a popular branch of mathematics related to computer science, Fractal geometry.

The talk presents some evidence for a close relationship between Kolmogorov complexity and information-like –or rather, entropy-like– measures investigated in Fractal geometry. We consider, as mentioned above, for infinite strings, the amount of information which must provided in order to specify a particular symbol of this sequence. This quantity can be measured using (the first order approximation of) the Kolmogorov complexity of infinite strings.

On the other hand, a set of infinite strings (ω-words) can be measured using entropy-like size-measures, so-called dimensions known from geometric measure theory or from the theory of dynamical systems. We consider here Minkowski (or box-counting) dimensions and the Hausdorff dimension.

It turns out that for computable sets of strings the mentioned dimension provide upper and lower bounds to the Kolmogorov complexity of maximally complex strings in the respective sets.

As computable sets of infinite strings we consider the following ones:

1. ω-languages of the first levels of the Arithmetical hierarchy
2. ω-languages defined by finite automata, so-called regular ω-languages
3. ω-languages defined as infinite products of recursive or recursively enumerable languages, so-called ω-power languages
4. ω-languages having certain combinatorial growth properties

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Inequalities on Divisors of Integer Polynomials

Doru Ştefănescu
The University of Bucharest
Romania
E-mail: `stef@zbl.imar.ro`

We obtain new inequalities on divisors of integer polynomials. Our results improve an inequality on Bombieri's l_2–weighted norm. We also obtain limits for the smallest divisor of an integer polynomial. In particular such bounds are very useful for algorithms of factorization of integer polynomials.

For obtaining refined inequalities we look not only to extremal coefficients in a general inequality on Bombieri's norm, as done by B. Beauzamy, but also to an arbitrary coefficient of a divisor. This idea was successfully used by V. Gonçalves to improve the inequality of E. Landau between the measure and the quadratic norm.

Our results improve the following evaluations of B. Beauzamy and M. Mignotte and Ph. Glesser, respectively:

$$\mathrm{H}(Q) \le \frac{3^{3/4}\cdot 3^{n/2}}{2(\pi n)^{1/2}}[P]_2 \ , \tag{1}$$

$$b(P) \le \left(\max_{1\le d\le n/2}\left\{\binom{d}{\lfloor d/2\rfloor}\binom{(n-d)}{\lfloor (n-d)/2\rfloor}\right\}\right)^{1/2} \mathrm{M}(P)^{1/2} \ , \tag{2}$$

where $[P]_2 = \sqrt{\sum_{j=0}^{n}|a_j|^2/\binom{d}{j}}$ is Bombieri's weighted l_2–norm, $\mathrm{H}(P) = \max\{|a_0|,|a_1|,\ldots,|a_n|\}$ is the height, $\mathrm{M}(P) = \exp\left\{\int_0^1 \log\big|P(e^{2i\pi t}\big|dt\right\}$ and $\int_0^1 \log\big|P(e^{2i\pi t}\big|dt$ are measures, and $b(P) = \min_i\{\mathrm{H}(Q_i)\,;\, Q_i \text{ is a divisor of } P\}$.