Neural Networks

Computer Science 367

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Some History of Neural Networks

- McCulloch and Pitts [1943]: Model of artificial neurons
- Hebb [1949]: Simple updating rule
- Minsky and Edmonds [1951]: First neural network computer
- Rosenblatt [1962]: Perceptrons (the model)
- Minsky and Papert [1969]: Perceptrons (the book)

Revival of Neural Networks

Recently, there has been a resurgence of interest in neural networks for the following reasons:

- Faster digital computers to simulate larger networks
- Interest in building massively parallel computers
- New neural network architectures
- Powerful Learning algorithms

Characteristics of Neural Networks

- A large number of very simple neuronlike processing elements
- A large number of weighted connections between the elements
- Highly parallel, distributed control
- An emphasis on learning internal representations automatically

The 100-Time-Steps Requirement

- Individual neurons are extremely slow devices (compared to digital computers), operating in the millisecond range.

- Yet, humans can perform extremely complex tasks in just a tenth of a second.

- This means, humans do in about a hundred steps what current computers cannot do in 10 million steps.

- Look for massively parallel algorithms that require no more than 100 time steps.

Failure Tolerance

- On the one hand, neurons are constantly dying, and their firing patterns are irregular

- On the other hand, components in digital computers must operate perfectly.

- With current technology, it is:
  - Easier to build a billion-component IC in which 95% of the components work correctly.
  - More difficult to build a million-component IC that functions perfectly.

Fuzziness

- People seem to be able to do better than computers in fuzzy situations.

- We have very large memories of visual, auditory, and problem-solving episodes.

- Key operation in solving new problems is finding closest matches to old situations.

Hopfield Networks

Theory of memory

- Hopfield introduced this type of neural network as a theory of memory.

Distributed representation

- A memory is stored as a pattern of activation across a set of processing elements.
- Furthermore, memories can be superimposed on one another; different memories are represented by different patterns over the same set of processing elements.

Hopfiled Networks (cont'd)

Distributed, asynchronous control

- Each processing element makes decisions based only on its own local situation. All the local actions add up to a global solution.

Content-addressable memory

- A number of patterns can be stored in a network. To retrieve a pattern, we need only specify a portion of it. The network automatically finds the closest match.

Fault tolerance

- If a few processing elements misbehave or fail completely, the network will still function properly.

Technical Details of Hopfield Networks

- Processing elements (units) are either in state active (1) or passive (-1).

- Units are connected to each other with weighted, symmetric connections (recurrent network).

- A positively (negatively) weighted connection indicates that the two units tend to activate (deactivate) each other.

Parallel Relaxation in Hopfield Networks

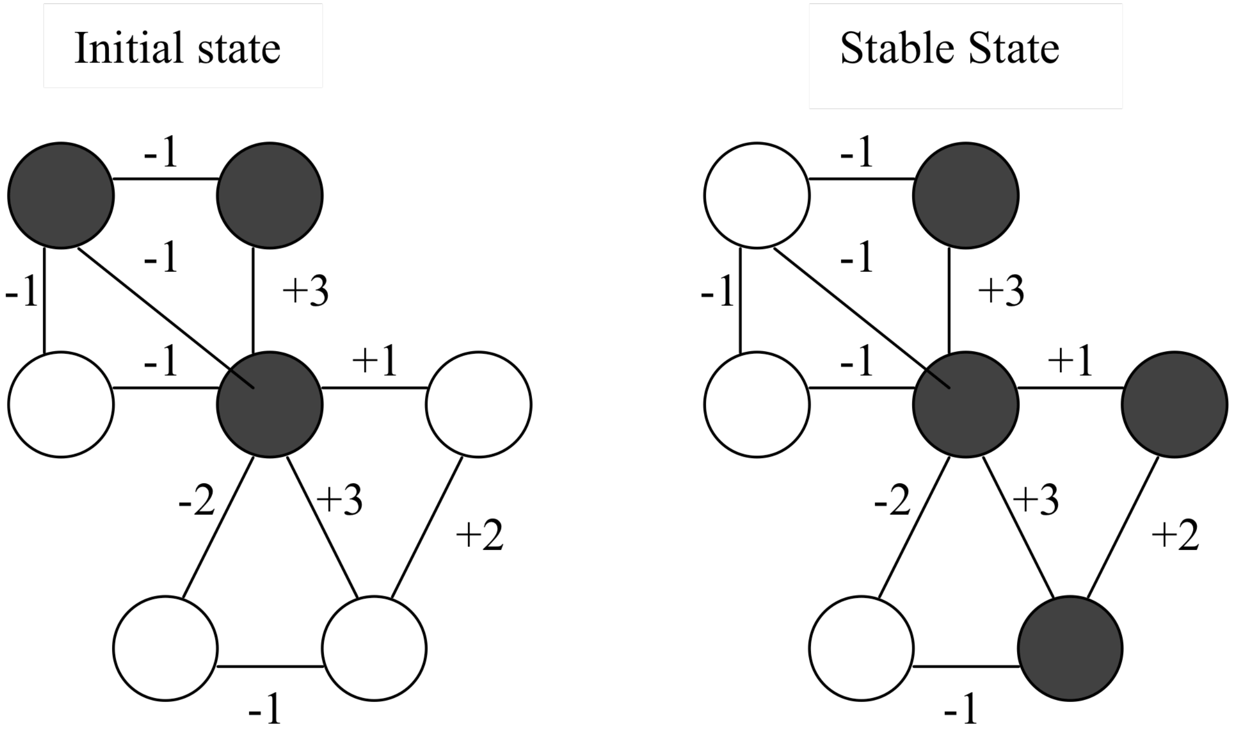
- A random unit is chosen.

- If any of its neighbors are active, the unit computes the sum of the weights on the connections to those active neighbors.

- If the sum is positive, the unit becomes active; otherwise it becomes inactive.

- The process (parallel relaxation) is repeated until the network reaches a stable state.

Example of a Hopfield Network



Initial state

Stable State

Stability

- Given any set of weights and any initial state, parallel relaxation eventually steers the network into a stable state.

- It will only stabilize if parallel relaxation is used (asynchronous).
- If a synchronous update is done then it will either stabilize or oscillate between two (and only two) units.

Some Features of Hopfield Networks

- The network can be used as a content-addressable memory by setting the activities of the units to correspond to a partial pattern. To retrieve the pattern, we need only supply a portion of it.

- Parallel relaxation is nothing more than search, albeit of a different style. The stable states correspond to local minima in the search space.

- The network corrects errors: if the initial state contains inconsistencies, the network will settle into the solution that violates the fewest constraints offered by the inputs.

Perceptrons

- This type of network was invented by Rosenblatt [1962].

- A perceptron models a neuron by taking a weighted sum of its inputs and sending the output 1 if the sum is greater than or equal to some adjustable threshold value; otherwise it sends 0.

- The connections in a perceptron, unlike in a Hopfield network, are unidirectional (feedforward network).

- Learning in perceptrons means adjusting the weights and the threshold.

- A perceptron computes a binary function of its input. Perceptrons can be combined to compute more complex functions.

A Perceptron

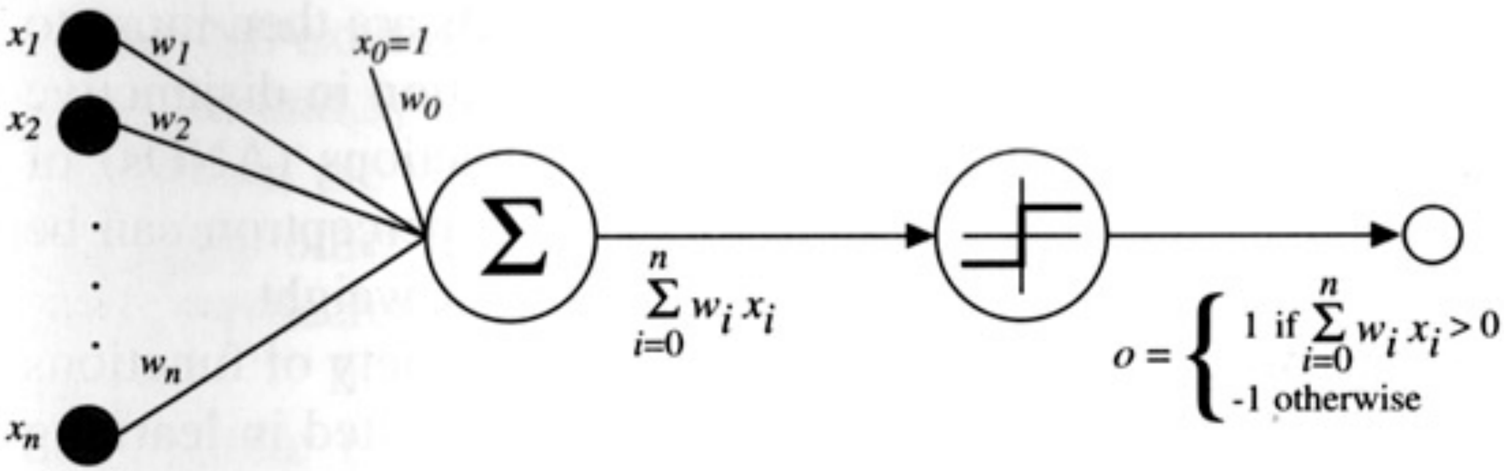

x_1
w_1
$x_0=1$
w_0
x_2
w_2
Σ
w_n
x_n
$\sum_{i=0}^{n} w_i x_i$
$o = \begin{cases} 1 \text{ if } \sum_{i=0}^{n} w_i x_i > 0 \\ -1 \text{ otherwise} \end{cases}$

FIGURE 4.2
A perceptron.

Activation Function

- Input: $\vec{x} = (x_1, \ldots, x_n) \qquad x_0 = 1$
- Output with explicit threshold:

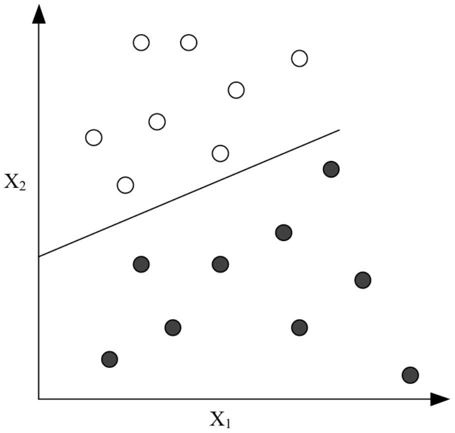
$$g(\vec{x}) = \begin{cases} 1 & if \; \sum_{i=1}^{n} w_i x_i \geq t \\ 0 & otherwise \end{cases}$$

- Output with implicit threshold:

$$g(\vec{x}) = \begin{cases} 1 & if \; \sum_{i=0}^{n} w_i x_i \geq 0 \\ 0 & otherwise \end{cases}$$

What Perceptrons Can Represent

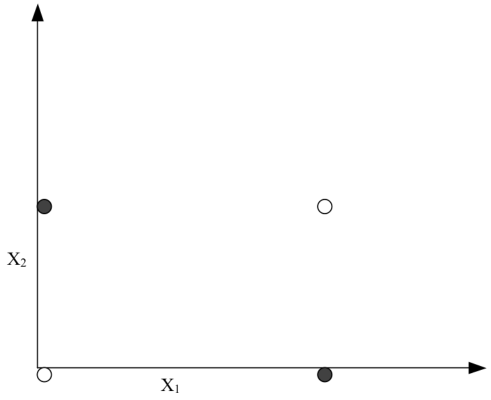
- Linearly Separable Function



- Input: (x_1, x_2)
- Output: $g(\vec{x}) = w_0 + w_1 x_1 + w_2 x_2$
- Decision Surface: $g(x_1, x_2) = 0 \Leftrightarrow x_2 = -\frac{w_1}{w_2} x_1 - \frac{w_0}{w_2}$

Limitations of Perceptrons

- If a decision surface does not exist, the perceptron cannot learn the function.
- An example is the XOR function:



Perceptron Learning Method

- Start with randomly assigned weights.

- For each example $\vec{x}$ do:
  - Let *o* be the computed output $g(\vec{x})$
  - Let *t* be the expected (target) output.
  - Update the weights based on $\vec{x}$, *o*, and *t*.

- Repeat the process (i.e., go through another epoch) until all examples are correctly predicted or the stopping criterion is reached.

Updating Rule

- The error is the difference between the expected output and the computed output:

$$err = t - o$$

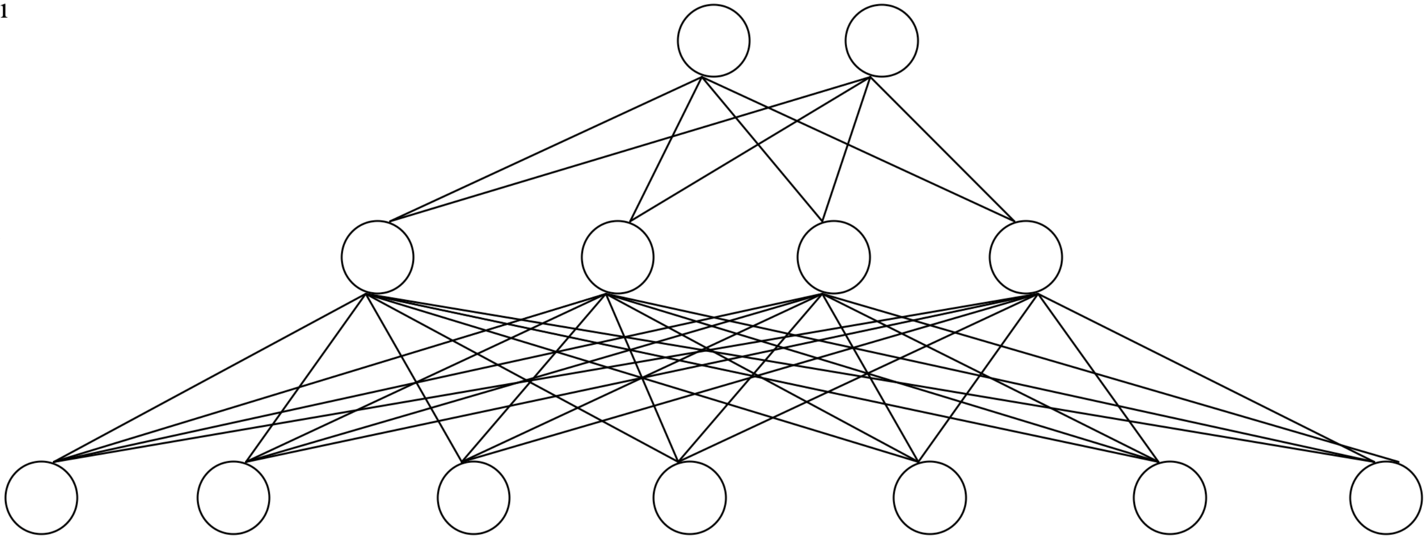
- If the error is positive (negative), *o* must be increased (decreased).
- Each input x_i contributes $w_i x_i$ to the total input.
- If x_i is positive (negative), increasing w_i will increase (decrease) *o*.
- The desired effect can be achieved with the following rule (η is the learning rate):

$$w_i \leftarrow w_i + \eta \cdot x_i \cdot err$$

Multilayer Feed-Forward Networks

- Input units are connected to hidden units.
- Hidden units are connected to other hidden units.
- . . .
- Hidden units are connected to output units.

Example of a Three Layer Feed-Forward Network



Output units O_i

$W_{j,i}$

Hidden units a_j

$W_{k,j}$

Input units I_k

The Idea of Back-Propagation Learning

- Compute the output for a given input and compare it with the expected output.

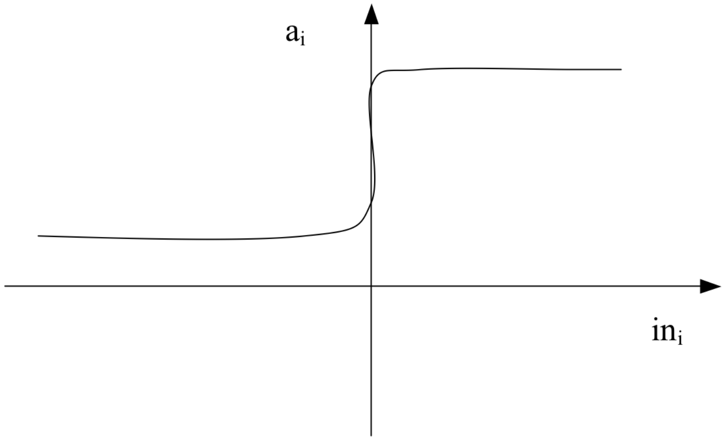
- Assess the blame for an error and divide it among the contributing weights.

- Start with the second layer (hidden units to output units) and then continue with the first layer (input units to hidden units).

- Repeat this for all examples and for as many epochs as it takes for the network to converge.

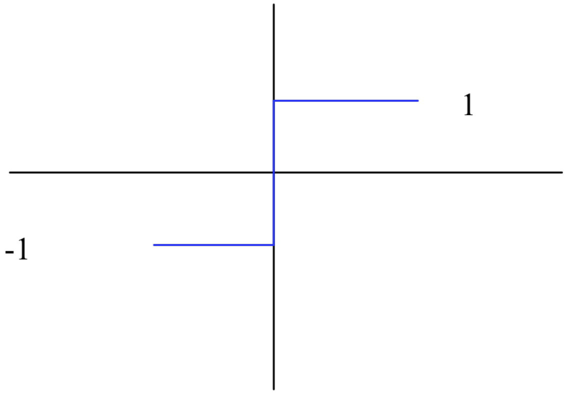
Activation Function

- Backpropagation requires the derivative of the activation function *g*.
- The sign function (used in Hopfield networks) and the step function (used in Perceptrons) are not differentiable.
- Usually, backpropagation networks use the sigmoid function:

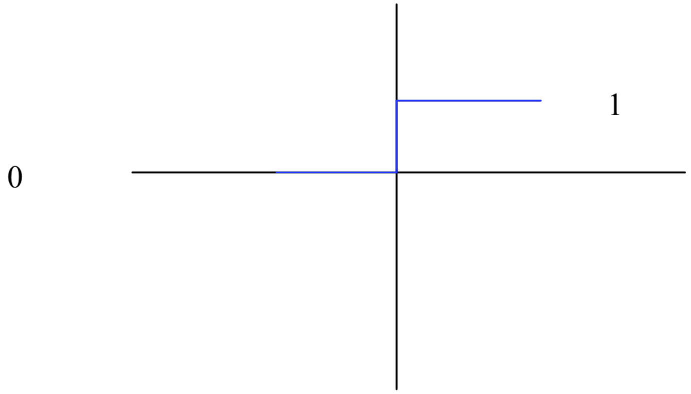


Sign And Step Functions

Sign Function



Step Function



Backpropagation Update Rules (2^{nd} Layer)

- Let Err_i be the error $(t_i - o_i)$ at the output node.

- Let in_i be the weighted sum $\sum_j w_{j,i} a_j$ of inputs to unit *i*.

- Let Δ_i be the new error term $Err_i\ g'(in_i)$.

- Then the weights in the second layer are updated as follows:

$$w_{j,i} \leftarrow w_{j,i} + \eta \cdot a_j \cdot \Delta_i$$

Backpropagation Update Rules (1st Layer)

- Let Δ_j be the new error term for the first layer:

$$\Delta_j = g'(in_j) \sum_i w_{j,i} \Delta_i$$

- Then the weights in the first layer are updated as follows:

$$w_{k,j} \leftarrow w_{k,j} + \eta \cdot i_k \cdot \Delta_j$$

Pros and Cons of Backpropagation

- Cons
  - Backpropagation might get stuck in a local minimum that does not solve the problem.
  - Even for simple problems like the XOR problem, the speed of learning is slow.
- Pros
  - Fortunately, Backpropagation does not get stuck very often,
  - Backpropagation is inherently a parallel, distributed algorithm.