

THE UNIVERSITY OF AUCKLAND

FIRST SEMESTER, 2008
Campus: Tamaki

COMPUTER SCIENCE

Algorithms and Data Structures

(Time allowed: ONE hour)

NOTE: Attempt *all* questions!

Put the answers on the Teleform sheet provided.

Ensure that you enter the correct version number for this exam.

Use a **dark pencil** only and to answer, fill in the appropriate box on the Teleform sheet.

If necessary, erase *completely* to change.

Do not fold, or write on the back of this form.

This is Version 1-4.

Use of calculators is NOT permitted.

<i>Section:</i>	A	B	C	Total
<i>Possible marks:</i>	20	20	20	60
<i>Awarded marks:</i>				

SURNAME:

FORENAME(S):

STUDENT ID:

CONTINUED

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Section A: Algorithm analysis

1. An algorithm has processing time $T(n) = 0.01n^5 + 3n^4 + 20n^3 + 300n^2 + 70n + 4$ for the input data size n . What is the “Big-Theta” time complexity of this algorithm? [Marks: 2]

$$\Theta(n^5)$$

2. Let the function $f(n)$ be $\Theta(n)$ (“Big-Theta” of n) and let the function $g(n)$ be $O(n \log n)$ (“Big-Oh” of $n \log n$). Then the product of these functions, i.e. the function $h(n) = f(n) \cdot g(n)$, is ... [Marks: 5]

$$O(n^2 \log n)$$

3. According to Transitivity Rule, if the function $h(n)$ is $O(g(n))$ and the function $g(n)$ is $O(f(n))$, then ... [Marks: 4]

$$h(n) \text{ is } O(f(n))$$

4. According to Rule of Sums, if the function $g_a(n)$ is $\Theta(f_a(n))$ and the function $g_b(n) = \Theta(f_b(n))$, then their sum $g_a(n) + g_b(n)$ is ... [Marks: 5]

$$\Theta(\max\{f_a(n), f_b(n)\})$$

5. The function $g(n)$ is $\Omega(f(n))$ (“Big Omega” of the function $f(n)$) if and only if there exists a positive real constant c and a positive integer n_0 such that ... [Marks: 4]

$$g(n) \geq cf(n) \text{ for all } n \geq n_0$$

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Section B: Basic recurrences, sorting and searching

6. For $n = 2^m$ with an integer $m > 0$, the recurrence $T(n) = 2T(n/2) + 1$ with the base case $T(1) = 0$ has a closed-form solution being equal exactly to ... [Marks: 5]

$$n - 1$$

7. The average time complexity of algorithm `insertion sort` is ... [Marks: 2]

$$\Theta(n^2)$$

8. The worst-case time complexity of algorithm `quicksort` is ... [Marks: 3]

$$\Theta(n^2)$$

9. Let the maximum heap have seven keys, [70, 50, 60, 20, 30, 40, 10], associated with its successive nodes. After the maximum key is deleted from this heap and the heap order is restored, what key will be associated with the right child of the root? [Marks: 5]

$$40$$

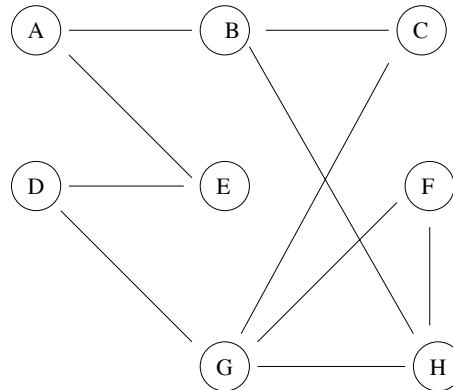
10. Let $h(k) = k \% 7$ be the hash function for a hash table T of size 7. After the items with the search keys 10, 21, 16, and 36 have been stored one-by-one in T using open addressing with linear probe, in which position the next item with the key 52 will be stored? [Marks: 5]

$$T[6]$$

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Section C: Graph algorithms



11. Consider the graph Γ shown. Its order is:

[Marks: 2]

8

12. Which of the following conditions can never be satisfied by a graph?

[Marks: 4]

- (a) Has diameter 4 and girth 6
- (b) Has 4 vertices and the distance between every pair of vertices is 1
- (c) Has 100 vertices and 99 edges, and is connected
- (d) Has 4 vertices and 6 edges
- (e) All the conditions are possible

All the conditions are possible

13. “The length of the shortest cycle of Γ ” is the definition of which attribute of a graph Γ ? [Marks: 2]

girth

14. Suppose that we carry out the DFS algorithm on a digraph Γ , and the following timestamps are recorded.

Node	0	1	2	3	4	5	6
Seen	0	1	2	11	4	3	6
Done	13	10	9	12	5	8	7

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Which of the following: (0, 3), (5, 4), (4, 5), (0, 1), and (5, 6) is NOT a tree arc in the DFS forest?

[Marks: 4]

(4, 5)

15. Suppose that we carry out the DFS algorithm on a digraph Γ , and the following timestamps are recorded.

Node	0	1	2	3	4	5	6
Seen	0	1	2	11	4	3	6
Done	13	10	9	12	5	8	7

If (1, 6) were an arc of Γ , which type of arc would it be?

[Marks: 3]

forward

16. Consider the digraph on nodes 0, 1, 2, 3, 4, 5, 6 with adjacency lists representation

2		
0		
0	1	
4	5	6
5		
3	4	6
1	2	

If we carry out DFS, using the rule that whenever we have a choice of node to visit, we choose the one with lowest label, how many trees are in the DFS forest?

[Marks: 5]

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