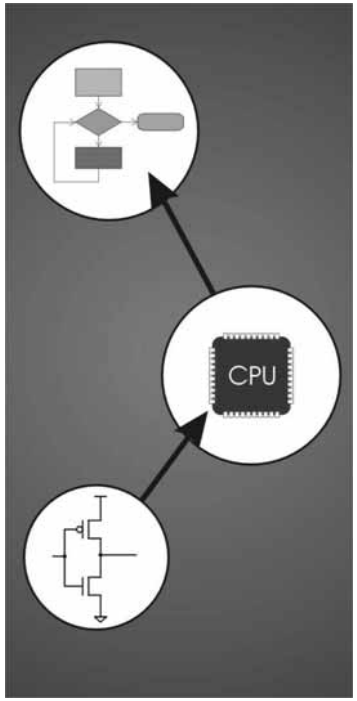

COMPSCI 210

Binary Fractions

Agenda & Reading

Topics:

- Fractions
  - Binary
  - Octal
  - Hexadecimal
- Conversion
  - Binary -> Octal, Hex
  - Octal -> Binary, Hex
  - Decimal -> Octal, Hex
  - Hex -> Binary, Octal

Animation:

- BinFrac.htm

Example Code:

- Frac2Bin.html

Exercise:

- Exercise 06-07

Fractions

Decimal

- A decimal fraction is a fraction where the denominator is a power of ten.
  - But are commonly expressed without a denominator,
  - e.g., 8/10, 833/100, 83/1000, and 8/10000 are expressed as: 0.8, 8.33, 0.083, and 0.0008
- $8.33 = 8 \times 10^{(0)} + 3 \times 10^{(-1)} + 3 \times 10^{(-2)}$
- $0.195 = 1 \times 10^{(-1)} + 9 \times 10^{(-2)} + 5 \times 10^{(-3)}$

Binary

- Proceeds in exactly the same way as decimal fractions
- Each place to the right of the decimal point represents a negative power of 2
- $0.11_2 = 1 \times 2^{(-1)} + 1 \times 2^{(-2)} = 0.75_{10}$
- $1.101_2 = 1 \times 2^{(0)} + 1 \times 2^{(-1)} + 0 \times 2^{(-2)} + 1 \times 2^{(-3)} = 1.625_{10}$

Shockwave Movie: BinFrac.htm

Conversion

Converting Binary to Decimal

- Method 1: Successively multiply the fraction by the outgoing base (e.g. decimal = 1010). At each stage take the integral part as the digit and retain the fraction for the next stage. BinFrac.htm
  - Example: 0.0101_2
    - $0.0101_2 * 1010 = 11.0010$ (Integral part =3)
    - $0.0010_2 * 1010 = 1.0100$ (Integral part =1)
    - $0.0100_2 * 1010 = 10.1000$ (Integral part =2)
    - $0.1000_2 * 1010 = 101.0000$ (Integral part =5)
    - = 0.3125
- Method 2: Working from the least significant fractional digit, append the digit as an integer prefix to the fraction-so-far and divide the whole by the incoming base (e.g. decimal=2)
  - Example: 0.101_2 (fraction-so-far=0)
    - Take bit 1, f-s-f = (0 + 1)/2 = 0.5
    - Take bit 0, f-s-f = (0.5 + 0)/2 = 0.25
    - Take bit 1, f-s-f = (0.25 + 1)/2 = 0.625

Shockwave Movie: BinFrac.htm

Conversion

Converting Decimal to Binary

- Take the fraction part multiply by 2, and convert the integral part to get the value
- Example: 0.25_{10}
  - Take 0.25 * 2 = 0.5 (Integral part = 0)
  - Take 0.5 * 2 = 1.0 (Integral part = 1)
  - = 0.01_2
- Exercise: 0.625_{10}
- However, There are some fractions we can't represent.0.3_{10}
  - Take 0.3 * 2 = 0.6 (Integral part = 0)
  - Take 0.6 * 2 = 1.2 (Integral part = 1)
  - Take 0.2 * 2 = 0.4 (Integral part = 0)
  - Take 0.4 * 2 = 0.8 (Integral part = 0)
  - Take 0.8 * 2 = 1.6 (Integral part = 1)
  - Take 0.6 * 2 = 1.2 (Integral part = 1)
  - = 0.0100110011001…

Repeated

Octal & Hexadecimal Fractions

Octal

- Each place to the right of the decimal point represents a negative power of 8
- $0.11_8 = 1 \times 8^{(-1)} + 1 \times 8^{(-2)} = 0.125_{10} + 0.015625_{10}$
- $=0.140625_{10}$

Weights	$8^{(2)}$	$8^{(1)}$	$8^{(0)}$	$8^{(-1)}$	$8^{(-2)}$
Weight Value	64	8	1	0.125	0.015625

Hexadecimal

- Each place to the right of the decimal point represents a negative power of 16
- $0.1_{16} = 1 \times 16^{(-1)} = 0.0625_{10}$

Weights	$16^{(2)}$	$16^{(1)}$	$16^{(0)}$	$16^{(-1)}$
Weight Value	256	16	1	0.0625

Examples

- $0.1_2 = 0.5_{10} = 0.4_8 = 0.8_{16}$
- $0.11_2 = 1 \times 2^{(-1)} + 1 \times 2^{(-2)} = 0.75_{10}$
  - $= 0.6_8 = 6 \times 8^{(-1)}$
  - $= 0.C_{16} = 12 \times 16^{(-1)}$

Conversion

Converting Binary to Octal

- Break the binary digits into groups of THREE starting from the binary point and convert each group into its appropriate octal digit
  - Whole numbers
    - it may be necessary to add a zero as the Most Significant Bit in order to complete a grouping of three bits.
  - Fractions,
    - it may be necessary to add a trailing zero in the Least Significant Bit in order to form a complete grouping of three.
- Example: 0.0101_2
  - $0.010\ 1\underline{00} = 0.24_8$

Converting Binary to Hexadecimal

- Break the binary digits into groups of FOUR starting from the binary point and convert each group into its appropriate hex digit. Again, it may be necessary to add leading/trailing zeros
- Example: 0.010101_2
  - $0.0101\ 01\underline{00} = 0.54_{16}$

Conversion

Converting Octal to Binary

- Replace the octal digit with the corresponding binary pattern
- Example: 0.142_8
  - $= 0.001\ 100\ 010_2$
- Exercise: 15.123_8

Converting Octal to Hexadecimal

- Replace the octal digit with the corresponding binary pattern, regroup digits in a group of 4 and replace with the corresponding hex digit
- Example: 2.47_8
  - $= 010.100\ 111_2$
  - $= 0010.\ 1001\ 11\underline{00}$
  - $= 2.9C_{16}$
- Exercise: 12.3_8

Conversion

```
Example:
Frac2Bin.html (Enter a decimal fraction
(0.0 <= value < 1.0), select the base
```

Converting Decimal to Octal

- Take the fraction part multiply by 8, and convert the integral part to get the value
- Example: 0.3125_{10}
  - Take 0.3125 * 8 = 2.5 (Integral part = 2)
  - Take 0.5 * 8 = 4.0 (Integral part = 4)
  - = 0.24_8
- Exercise: 0.625_{10}

Converting Decimal to Hexadecimal

- Take the fraction part multiply by 16, and convert the integral part to get the value
- Example: 0.625_{10}
  - Take 0.625 * 16 = 10.0 (Integral part = 10)
  - = $0.a_{16}$

Conversion

Converting Hexadecimal to Binary

- Replace the hex digit with the corresponding binary pattern
- Example: $2.A7_{16}$
  - = $0010.1010\ 0111_2$
- Exercise: 1.F316

Converting Hexadecimal to Octal

- Replace the hex digit with the corresponding binary pattern, regroup digits in a group of 3 and replace with the corresponding octal digit
- Example: $2.A7_{16}$
  - = $0010.1010\ 0111_2$
  - = 010. 101 001 11<u>0</u>
  - = 2.516_8
- Exercise : $1.F3_{16}$

Exercises:

- Convert 1.101011 from Binary Fraction to Decimal Fraction
- Convert 0.625 from decimal Fraction to Binary Fraction

Exercises:

- Convert octal fraction 15.123 to Binary fraction
- Convert octal fraction 12.3 to Hexadecimal fraction

Exercises:

- **Convert Hexadecimal fraction 1.F3 to Binary fraction**

- **Convert Hexadecimal fraction 1.F3 to Octal**

Binary Fractions – Addition & Subtraction

add/subtract fractions using simple Math (align the radix point)

- **Example:** 0.1010 + 0.010

```
 0.1010
+0.0100
 0.1110
```

- **Example:** 0.1010 - 0.1

```
 0.1010
-0.1000
 0.0010
```

- **Since we're just doing natural arithmetic, the carry flag has the same meaning**

```
 0.1010
+0.1000
 1.0010
```


Exercises

- **10.101 + 1.111**

- **1011.101 – 11.011**

Binary Fractions - Multiplication

Steps:

- **Ignore the radix point -- do not align them -- and**
- **multiply the numbers as whole numbers.**
- **Then, starting from the right of the product, separate as many places as there are in the two numbers together.**
- **Example: 10.01 * 1.01**

```
  1001
 * 101
------
  1001
 0000
1001
------
101101
```

- **Now we must put back the radix point. Together, 10.01 and 1.01 have four binary places. Therefore, starting from the right, separate four places: 10.1101**

Example:

```
      101.101     (A) (5.625 in decimal)
    ×  110.01     (B) (6.25  in decimal)
  -----------------
      101101    ← Corresponds to a one in B
 +    000000      ← Corresponds to a zero in B
 +   000000
 +  101101
 +101101
 ----------------------
=100011.00101     (35.15625 in decimal)
```


Binary Fractions - Division

Steps:

- The divisor must be a whole number. If it is not, then we will multiply (or shifting) it so that it becomes one.
- And Multiply (shifting) the dividend by the same power
- Example: 10.1101 / 1.01
- => 1011.01 / 101
- Answer = 10.01

```
            10.01
       ____________
101 | 1011.01
      101
        01
         0
        1 0
          0
         101
         101
           0
```


Exercises

- *1011.101 * 11.011*
- *1011.101 / 11.011*