An Example

Given

- the location of a point with respect to a reference frame O_{XYZ} attached to camera O,

  $$\mathbf{p} = [2,3,10]^T \text{ (in m)}$$

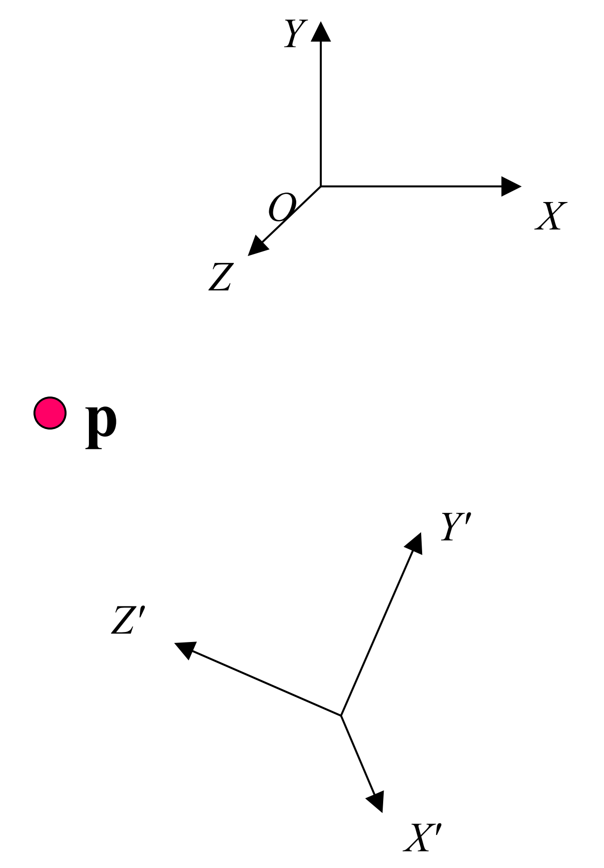
- Camera O' is 15m away in the direction of the –ve *y*-axis, and its RPY angles with respect to camera O are $(30^o,45^o,0^o)$ degrees.

$$\tilde{\mathbf{p}} \rightarrow \tilde{\mathbf{p}}'$$

Find

- The homogeneous transformation mapping.
- **p'** the location of the point in the reference frame $O'_{X'Y'Z'}$ attached to camera O'.

Schematic Diagram



- Determine $\mathbf{T} = \begin{bmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0}^{\mathrm{T}} & 1 \end{bmatrix}$
- The translation component is $\mathbf{t} = \begin{bmatrix} 0 \\ 15 \\ 0 \end{bmatrix}$
- What is the rotation matrix **R**?

- $\mathbf{R}(\theta_X, \theta_Y, \theta_Z) = \mathbf{R}_Z(\theta_Z)\, \mathbf{R}_Y(\theta_Y)\, \mathbf{R}_X(\theta_X)$

$$\mathbf{R}_X(\theta_X) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_X & -\sin\theta_X \\ 0 & \sin\theta_X & \cos\theta_X \end{bmatrix}$$

$$\mathbf{R}_Y(\theta_Y) = \begin{bmatrix} \cos\theta_Y & 0 & \sin\theta_Y \\ 0 & 1 & 0 \\ -\sin\theta_Y & 0 & \cos\theta_Y \end{bmatrix}$$

$$\mathbf{R}_Z(\theta_Z) = \begin{bmatrix} \cos\theta_Z & -\sin\theta_Z & 0 \\ \sin\theta_Z & \cos\theta_Z & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- $\mathbf{R}(30^o,45^o,0^o) = \mathbf{R}_Z(0^o)\ \mathbf{R}_Y(45^o)\ \mathbf{R}_X(30^o)$

$$\mathbf{R}_X(30^\circ) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 \\ 0 & 1/2 & \sqrt{3}/2 \end{bmatrix} \quad \mathbf{R}_Y(45^\circ) = \begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix}$$

$$\mathbf{R}_Z(\theta_Z) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- $\mathbf{R}(30^o,45^o,0^o) = \mathbf{R}_Z(0^o)\ \mathbf{R}_Y(45^o)\ \mathbf{R}_X(30^o)$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 \\ 0 & 1/2 & \sqrt{3}/2 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & 1/2\sqrt{2} & \sqrt{3}/2\sqrt{2} \\ 0 & \sqrt{3}/2 & -1/2 \\ -1/\sqrt{2} & 1/2\sqrt{2} & \sqrt{3}/2\sqrt{2} \end{bmatrix}$$

- So **T** is

$$\mathbf{T} = \begin{bmatrix} 1/\sqrt{2} & 1/2\sqrt{2} & \sqrt{3}/2\sqrt{2} & 0 \\ 0 & \sqrt{3}/2 & -1/2 & 15 \\ -1/\sqrt{2} & 1/2\sqrt{2} & \sqrt{3}/2\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Determine point in camera O'.

$$\tilde{\mathbf{p}}' = \mathbf{T}\,\tilde{\mathbf{p}}$$

$$\tilde{\mathbf{p}}' = \begin{bmatrix} 1/\sqrt{2} & 1/2\sqrt{2} & \sqrt{3}/2\sqrt{2} & 0 \\ 0 & \sqrt{3}/2 & -1/2 & 15 \\ -1/\sqrt{2} & 1/2\sqrt{2} & \sqrt{3}/2\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 10 \\ 1 \end{bmatrix} \approx \begin{bmatrix} 8.60 \\ 12.6 \\ 5.77 \\ 1 \end{bmatrix}$$

- so $\mathbf{p}' = [8.60, 12.6, 5.78]^T$ (metres)

Exercise

- What are the homogeneous representations for 3D points:
  a) with coordinates (10,14,0).
  b) on the x-axis 10 away from the origin.
  c) on the z-axis at infinite.

Exercise

- What *in*homogeneous points do these *homogeneous* coordinates represent?

$$\tilde{\mathbf{p}} = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 1 \end{bmatrix} \qquad \tilde{\mathbf{p}} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \qquad \tilde{\mathbf{p}} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$\tilde{\mathbf{p}} = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix} \qquad \tilde{\mathbf{p}} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \qquad \tilde{\mathbf{p}} = \begin{bmatrix} 2 \\ 6 \\ f \end{bmatrix}$$