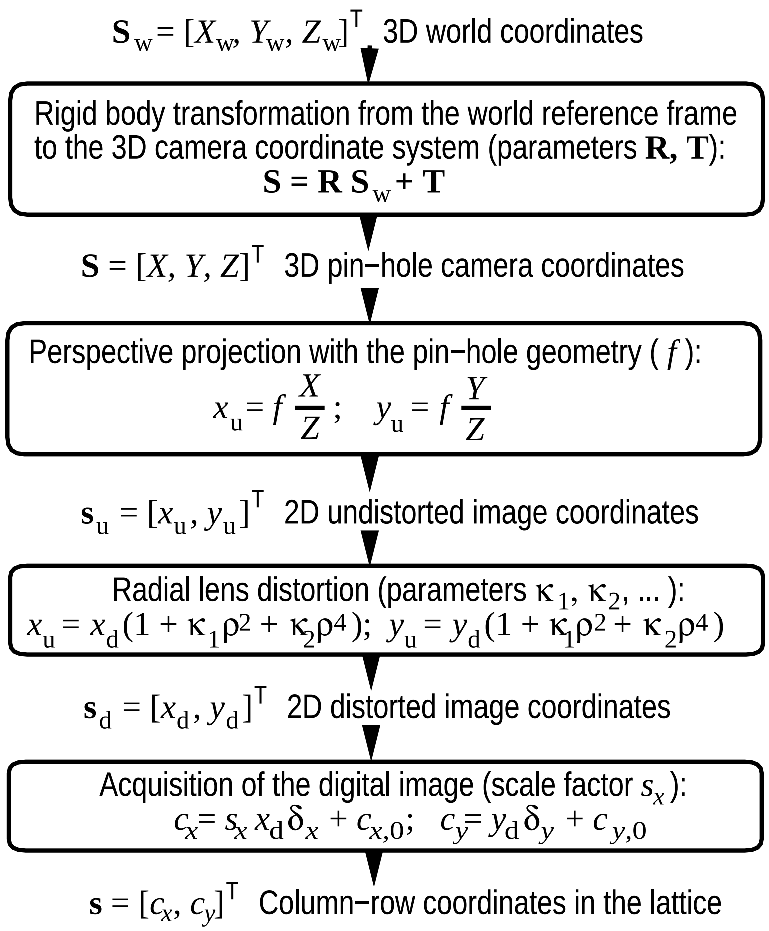
COMPSCI 773 S1 T – 2008

Camera Calibration

Tsai's calibration



Calibration steps 1 − 2

The Tsai's method requires at least 7 non-coplanar 3D reference points. The scale s_x is initially set to 1 and will be determined at step 4.

1. Find coordinates $[x_\mathsf{d}, y_\mathsf{d}]$ in the distorted image from the lattice coordinates $[c_x, c_y]$ assuming that $[c_{x,0}, c_{y,0}]$ is the image center:

$$x_\mathsf{d} = \frac{\delta_x(c_x - c_{x,0})}{s_x}; \quad y_\mathsf{d} = \delta_y(c_y - c_{y,0}).$$

2. Find 7 parameters of transforming image coordinates into world coordinates.

   (a) Exclude the denominator from

$$\frac{x_\mathsf{d}}{s_x} = \frac{r_{11}X_w + r_{12}Y_w + r_{13}Z_w + t_x}{r_{31}X_w + r_{32}Y_w + r_{33}Z_w + t_z}$$

$$y_\mathsf{d} = \frac{r_{21}X_w + r_{22}Y_w + r_{23}Z_w + t_y}{r_{31}X_w + r_{32}Y_w + r_{33}Z_w + t_z}$$

   (b) Form the linear equation

$$\left(x_\mathsf{d} = \mathbf{m}^\mathsf{T}\mathbf{L}\right)$$

assuming that

$$\mathbf{m} = [y_d X_w, y_d Y_w, y_d Z_w, y_d, -x_d X_w, -x_d Y_w, -x_d Z_w]^\mathsf{T};$$

$(t_y \neq 0)$:

$$\mathbf{L} = \frac{1}{t_y}[s_x r_{11}, s_x r_{12}, s_x r_{13}, s_x t_x, r_{21}, r_{22}, r_{23}]^\mathsf{T}.$$

(c) More than 7 reference points lead to an over-determined system of equations

$$x_{d,i} = \mathbf{m}_i^\mathsf{T}\mathbf{L}; \quad i = 1, \ldots, N$$

or $\mathbf{X} = \mathbf{ML}$ where $\mathbf{X}$ is the $N \times 1$ vector of $x_{d,i}$ values and $\mathbf{M}$ is the $N \times 7$ matrix with the row vectors $\mathbf{m}_i^\mathsf{T}$.

This system can be solved by using the pseudo-inverse technique (*Moore-Penrose inverse*):

$$\mathbf{L} = \left(\mathbf{M}^\mathsf{T}\mathbf{M}\right)^{-1}\mathbf{M}^\mathsf{T}\mathbf{X}$$

Moore-Penrose inverse

Let $\mathbf{Ax} = \mathbf{b}$ be an over-determined system of linear equations where $\mathbf{A}$ is a matrix $n \times m$, $\mathbf{x}$ is a vector $m \times 1$, and $\mathbf{b}$ is a vector $n \times 1$. The pseudo-inverse matrix for this system is obtained by solving the least-square problem $\min_{\mathbf{x}} D(\mathbf{x})$ where

$$D(\mathbf{x}) = ||\mathbf{Ax} - \mathbf{b}||^2 \equiv \mathbf{x}^\mathsf{T}\mathbf{A}^\mathsf{T}\mathbf{Ax} - 2\mathbf{x}^\mathsf{T}\mathbf{A}^\mathsf{T}\mathbf{b} + \mathbf{b}^\mathsf{T}\mathbf{b}.$$

Minimisation yields: $\frac{\partial D(\mathbf{x})}{\partial \mathbf{x}} = 0$, or

$$(\mathbf{A}^\mathsf{T}\mathbf{A})\mathbf{x} - \mathbf{A}^\mathsf{T}\mathbf{b} = 0$$

Thus the solution is: $\mathbf{x} = (\mathbf{A}^\mathsf{T}\mathbf{A})^{-1}\mathbf{A}^\mathsf{T}\mathbf{b}$ if the square $m \times m$ matrix $\mathbf{A}^\mathsf{T}\mathbf{A}$ is of rank m.

Calibration steps 3–4

1. Find the Y-coordinate of the translation vector using the parameter vector $\mathbf{L}$ obtained at step 2 and the orthonormality property of the rotation matrix $\mathbf{R}$:

$$r_{21}^2 + r_{22}^2 + r_{23}^2 = 1 \quad \rightarrow$$

$$|t_y| = \frac{1}{\sqrt{a_5^2 + a_6^2 + a_7^2}}$$

   where a_i denotes the i-th component of the parameter vector $\mathbf{L}$:

$$\mathbf{L} = [a_1, a_2, \ldots, a_7]^\mathsf{T}.$$

2. Determine the scaling factor s_x using the orthonormality property of $\mathbf{R}$:

$$s_x = |t_y| \sqrt{a_1^2 + a_2^2 + a_3^2}$$

3. Find the sign of t_y as follows.

   (a) Choose the reference 3D point whose image position is the most distant from the principal point (image center).

(b) Compute the parameters

$$r_{11} = a_1 t_y;\ r_{12} = a_2 t_y;\ r_{13} = a_3 t_y;$$
$$r_{21} = a_5 t_y;\ r_{22} = a_6 t_y;\ r_{23} = a_7 t_y;$$
$$t_x = a_4 t_y.$$

(c) Compare the signs

$$\text{sign}\{r_{11} X_w + r_{12} Y_w + r_{13} Y_w + t_x\};$$
$$\text{sign}\{r_{21} X_w + r_{22} Y_w + r_{23} Y_w + t_y\}$$

of the computed coordinates of the projected point to the signs of the actual image coordinates x, y. If the signs do not coincide, the sign of t_y should be inverted.

Calibration step 5

1. Recalculate the 6 components of the rotation matrix $\mathbf{R}$ and the X-component of the translation vector $\mathbf{t}$:

$$r_{11} = a_1 \frac{t_y}{s_x}; \quad r_{12} = a_2 \frac{t_y}{s_x}; \quad r_{13} = a_3 \frac{t_y}{s_x};$$
$$r_{21} = a_5 t_y; \quad r_{22} = a_6 t_y; \quad r_{23} = a_7 t_y;$$
$$t_x = a_4 \frac{t_y}{s_x}.$$

2. Calculate the remaining 3 components of the rotation matrix using the inner vector product of its first two rows:

$$r_{31} = \lambda \begin{vmatrix} r_{12} & r_{13} \\ r_{22} & r_{23} \end{vmatrix}; \quad r_{32} = \lambda \begin{vmatrix} r_{13} & r_{11} \\ r_{23} & r_{21} \end{vmatrix};$$

$$r_{31} = \lambda \begin{vmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{vmatrix}$$

where the factor λ is given by the orthonormality $r_{31}^2 + r_{32}^2 + r_{33}^2 = 1$.

Calibration step 6

Approximate the focal length f and the Z-coordinate of the translation vector taking no account of the lens distortion.

Projection relation for a reference point i:

$$y_{\mathrm{d},i} = \frac{r_{21}X_{w,i}+r_{22}Y_{w,i}+r_{23}Z_{w,i}+t_y}{r_{31}X_{w,i}+r_{32}Y_{w,i}+r_{33}Z_{w,i}+t_z}$$

is rewritten as the linear equation w.r.t. f and t_z:

$$[U_{y,i} \;\; -y_{\mathrm{d},i}] \left[\begin{array}{c} f \\ t_z \end{array} \right] = U_{z,i} y_{\mathrm{d},i}$$

where $U_{y,i} = r_{21}X_{w,i} + r_{22}Y_{w,i} + r_{23}Z_{w,i} + t_y$ and $U_{z,i} = r_{31}X_{w,i} + r_{32}Y_{w,i} + r_{33}Z_{w,i}$.

More than 2 reference points i result in the over-determined system of equations:

$$\mathbf{M} \left[\begin{array}{c} f \\ t_z \end{array} \right] = \mathbf{m}$$

to be solved by the pseudo-inverse technique where:

$$\mathbf{M}^{\mathsf{T}} = \left[\begin{array}{cccc} U_{y,1} & U_{y,2} & \cdots & U_{y,n} \\ -y_{\mathrm{d},1} & -y_{\mathrm{d},2} & \cdots & -y_{\mathrm{d},n} \end{array} \right]$$

and

$$\mathbf{m}^{\mathsf{T}} = \left[\begin{array}{cccc} U_{z,1} y_{\mathsf{d},1} & U_{z,2} y_{\mathsf{d},2} & \cdots & U_{z,n} y_{\mathsf{d},n} \end{array}\right].$$

The desired solution is as follows:

$$\left[\begin{array}{c} f \\ t_z \end{array}\right] = (\mathbf{M}^{\mathsf{T}}\mathbf{M})^{-1}\mathbf{M}^{\mathsf{T}}\mathbf{m}.$$

More accurate solutions are obtained with the steepest descent optimisation that starts from the already determined approximate values.