Dynamic Programming Stereo

COMPSCI 773 S1 T

VISION GUIDED CONTROL

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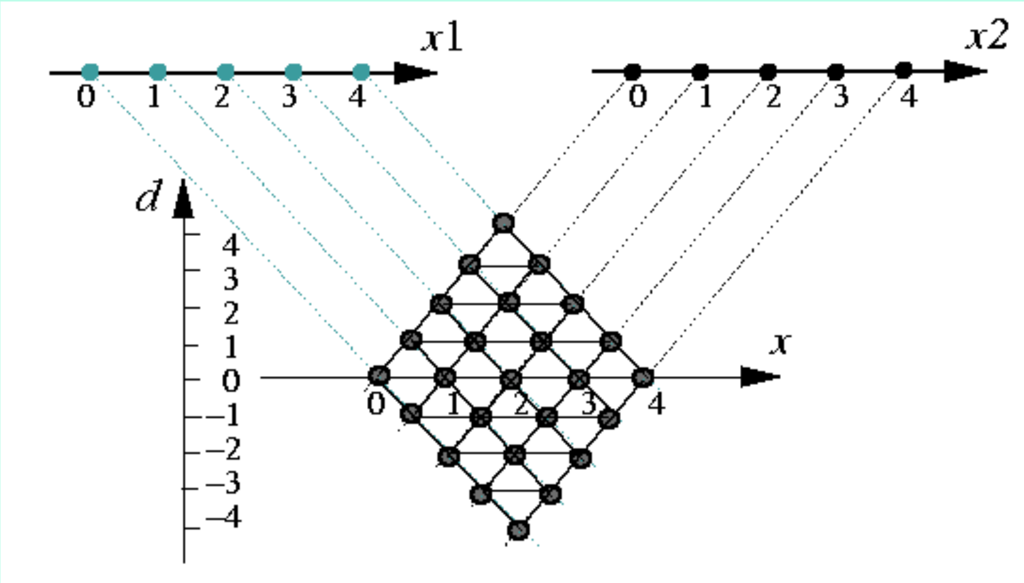
Markov Chain Model of a Profile

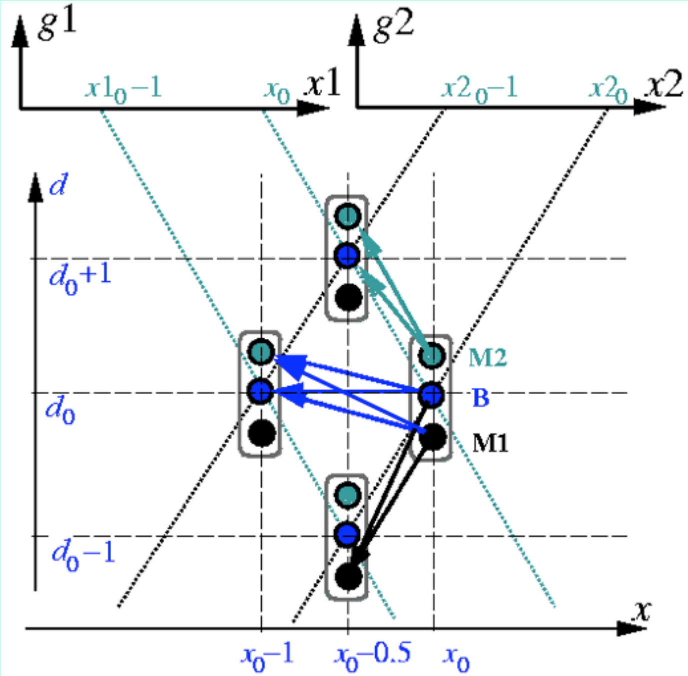
- Accounts for symmetry of stereo channels, visibility of 3-D points and discontinuities due to occlusions
- Single continuous surface only - the **ordering** constraint



Graph of profile variants (GPV)

Admissible Transitions

- Partial occlusions impose the **visibility constraint** on x-disparities in the symmetric coordinates (x,d):



$$d_{x,y} - 1 \le d_{x-0.5,y} \le d_{x,y} + 1$$

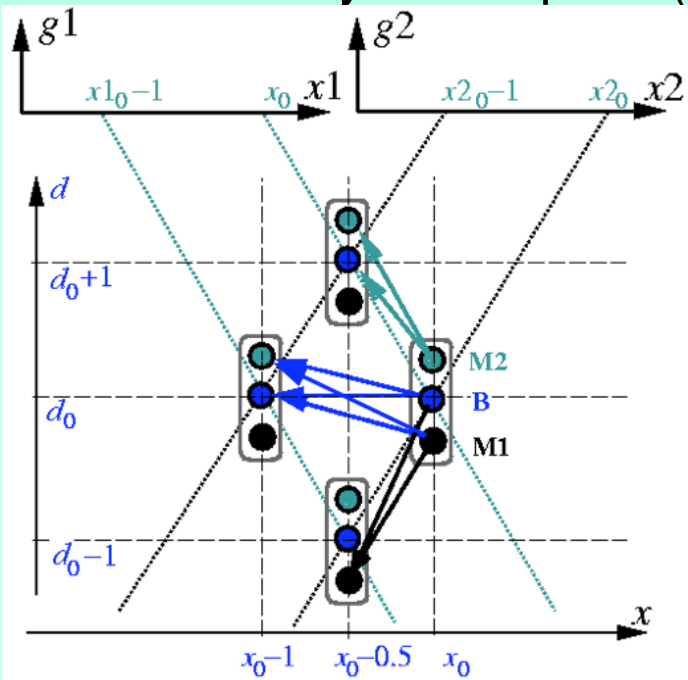
$$d_{x,y} - 2 \le d_{x-1,y} \le d_{x,y} + 2$$

Assumption: every 3-D point is visible either binocularly (B) or monocularly by the left (M1) or right (M2) camera

Transitions between the nodes in a GPV form admissible variants of an epipolar profile y

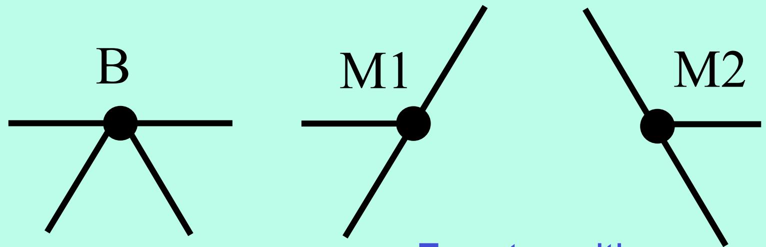
Admissible Transitions

Depending on the incoming and outcoming transition, the GPV node represents either a binocularly visible point (BVP), or a monocularly visible point (MVP) occluded in one image



Each node $\mathbf{v} = (x,d,s)$ has three visibility states s indicating the binocular, $s = \mathrm{B}$, or only monocular, $s = \mathrm{M1}$ or $\mathrm{M2}$, viewing

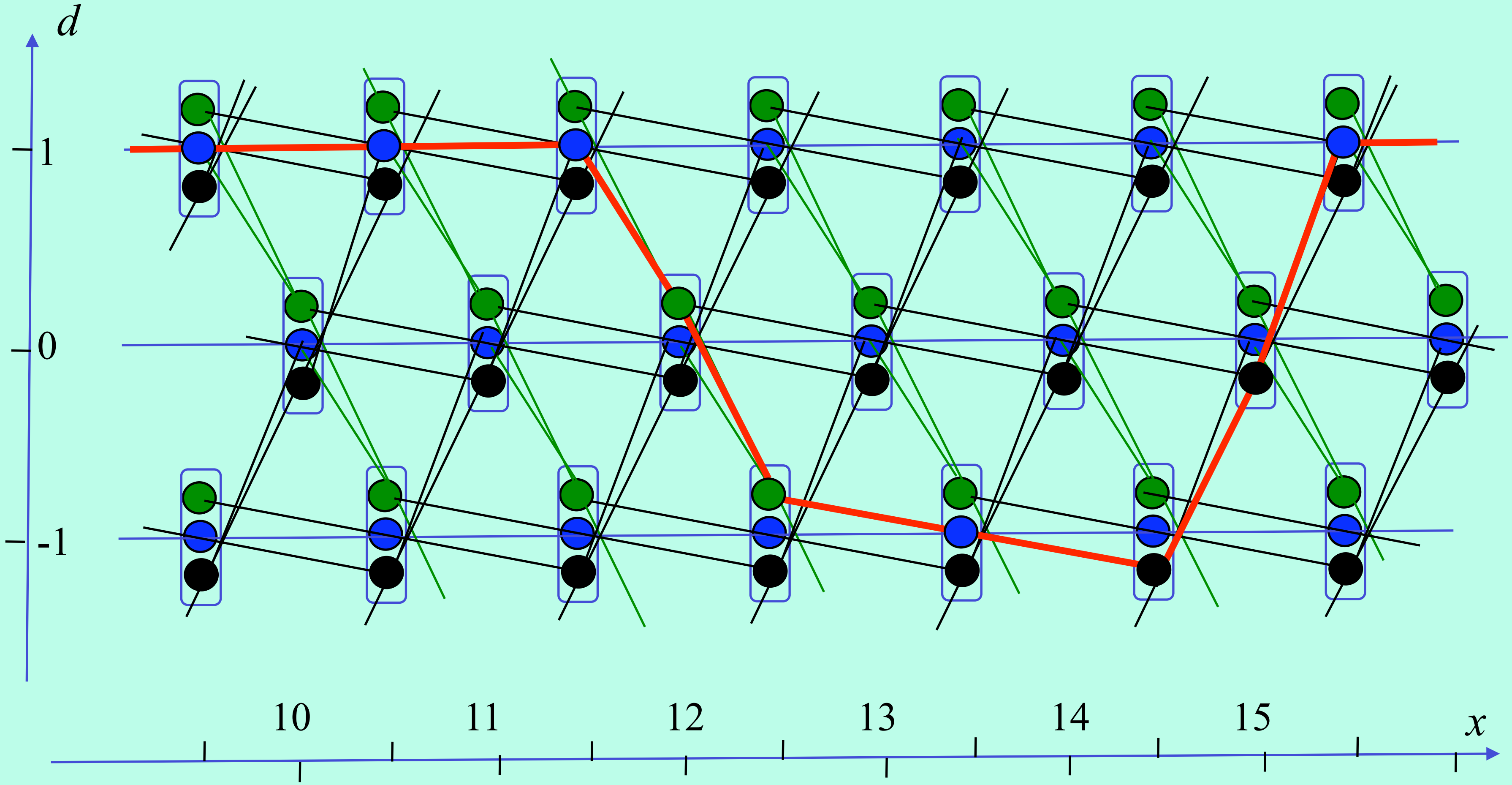
Only eight transitions are allowed in a GPV:



Four transitions forming a BVP

Four transitions forming a MVP

Admissible Transitions



Admissible Transitions

BVP ● :

$$(x,d,\mathrm{B}) \Rightarrow \left(g_{1:x+d/2}; g_{2:x-d/2}\right);$$

$$\left(g_{1:x_{pr}+d_{pr}/2}; g_{2:x_{pr}-d_{pr}/2}\right)$$

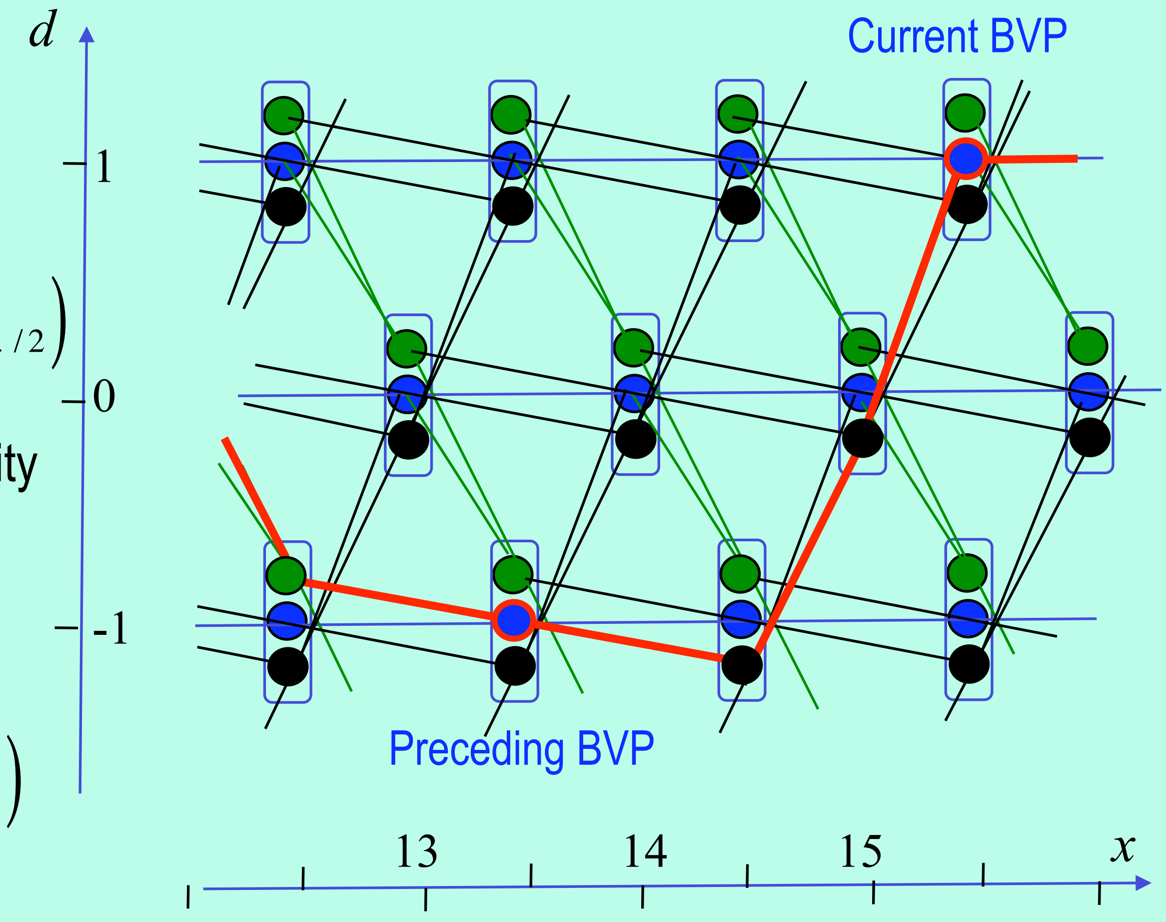
Pixel-wise signal dissimilarity

MVP ● ● :

$$(x,d,\mathrm{ML});\ (x,d,\mathrm{MR})$$

$$\Rightarrow \left(g_{1:x_{pr}+d_{pr}/2}; g_{2:x_{pr}-d_{pr}/2}\right)$$

Fixed “dissimilarity” weight



Markov Chain x-Profile Model

- Probability of a profile $\mathbf{d} = [(x_i, d_i, s_i): i = 1,\ldots,n]$:

$$\Pr\left(\mathbf{d} \middle| \mathbf{g}_L, \mathbf{g}_R\right) = p\left(x_1, d_1, s_1 \middle| \mathbf{g}_1, \mathbf{g}_2\right) \prod_{i=2}^{n} p\left(x_i, d_i, s_i \middle| x_{i-1}, d_{i-1}, s_{i-1}; \mathbf{g}_1, \mathbf{g}_2\right)$$

  where each term depends on transitions from the GVP-node in state s' to the node in state s'' along the profile

  – Transitions are limited by the visibility states along a GVP

  – Probability $p(x_i, d_i, \mathrm{B} | x_{i-1}, d_{i-1}, s_{i-1}; \mathbf{g}_1, \mathbf{g}_2)$ of a transition to state B depends on dissimilarity between the corresponding image signals for the current BVP on a profile's variant

    - It can also depend on the signals for the preceding BVP along this variant

  – Transition probabilities to the MVPs can be related to those for the BVPs

    - Typical simplification: a constant dissimilarity weight

Corresponding Signals Models

- Symmetric photometric model: signal distortions in the stereo images g_1 and g_2 w.r.t. the unobserved "ideal" cyclopean image (or ortho-image) g of a 3-D scene
- The simplest model: $g_{1:x_1,y_1} = g_{x,y} + n_{1:x,y}; \quad g_{2:x_2,y_2} = g_{x,y} + n_{2:x,y}$

Independent central-symmetric noise (monotone decrease of probability density)

Distortion-less signal for a 3-D point

- Model of contrast distortions - positive transfer factors, a, varying over the field of view of each camera and pixel-wise independent random noise, n, of image sensors:

$$g_{1:x_1,y_1} = a_{1:x,y}g_{x,y} + n_{1:x,y}; \quad g_{2:x_2,y_2} = a_{2:x,y}g_{x,y} + n_{2:x,y}$$

Model of Contrast Distortions

- <u>Transfer factors</u>: strong dependences for adjacent BVPs to account for visual resemblance of corresponding areas
- **Symmetric difference model** of the interdependence:
  - Limited direct proportion between the noiseless increments within a range: $0 < e_{\min} \le e_{\max}$ of difference factors e:

$$\min_{e_1 \in [e_{\min}, e_{\max}]} \left\{ e_1 \left(g_{x,y} - g_{x',y'} \right) \right\} \le a_{1:x,y} g_{x,y} - a_{1:x',y'} g_{x',y'} \le \max_{e_1 \in [e_{\min}, e_{\max}]} \left\{ e_1 \left(g_{x,y} - g_{x',y'} \right) \right\}$$

$$\min_{e_2 \in [e_{\min}, e_{\max}]} \left\{ e_2 \left(g_{x,y} - g_{x',y'} \right) \right\} \le a_{2:x,y} g_{x,y} - a_{2:x',y'} g_{x',y'} \le \max_{e_2 \in [e_{\min}, e_{\max}]} \left\{ e_2 \left(g_{x,y} - g_{x',y'} \right) \right\}$$

Neighbouring BVPs along the same epipolar profile

Model of Contrast Distortions

– Assuming that both the ortho-image signals g and random noise n are statistically independent, the difference model results in a Markov chain of signals for the successive BVPs and in the independent signals for the MVPs along a profile
  - Under a known 3-D profile, statistical estimates for the ortho-image g and transfer factors a_1, a_2 are derived on the basis of assumptions about the random noise and allowable spatial variations of the factors

– Theoretically justified part of a similarity measure is based on dissimilarity between the corresponding signals for the BVPs

– Heuristic regularising part of the measure relates to the MVPs
  - E.g. the assumptions of the links between the MVPs and near BVPs

Dynamic Programming Stereo

- Simplified notation: the y-coordinate is omitted
  - $\mathbf{d} = ((x_i, d_i, s_i) : i = 1,2,\ldots,N)$ - a digital profile as a sequence of the adjacent nodes in a GPV along allowable transitions

    $\mathbf{g} = (g_i$: i=1,2,…,N); $\mathbf{g}_1$= ($g_{1:i}$: i=1,2,…,N_1); $\mathbf{g}_2$= ($g_{2:i}$: i=1,2,…,N_2) - sequences of the cyclopean, left, and right image signals for the profile $\mathbf{d}$
  - For brevity: $g_i \equiv g_{x_i,y}$; $g_{1:i} \equiv g_{1:x_i+d_i/2,y}$; $g_{2:i} \equiv g_{2:x_i-d_i/2,y}$
- Signal model with only noise: $g_{1:i} = g_i + n_{1:i}$; $g_{2:i} = g_i + n_{2:i}$
- Signal model with varying contrast:

$$g_{1:i} = a_{1:i} g_i + n_{1:i}; \quad g_{2:i} = a_{2:i} g_i + n_{2:i}$$

- Signal model with varying contrast and offset:

$$\Delta g_{1:i,i-1} = e_{1:i} \Delta g_{i,i-1} + \Delta n_{1:i,i-1}; \quad \Delta g_{2:i,i-1} = e_{2:i} \Delta g_{i,i-1} + \Delta n_{2:i,i-1}$$

Pixel-wise Signal Dissimilarity

- The simplest symmetric signal model (only noise):
  - Grayscale signals and symmetric noise: $g_{1:i} = g_i + n_{1:i}; \quad g_{2:i} = g_i + n_{2:i}$

$$\min_{g_i}\left\{\max\left\{(g_{1:i} - g_i)^2, (g_{1:i} - g_i)^2\right\}\right\} \Rightarrow g_i = \frac{g_{1:i} + g_{2:i}}{2} \Rightarrow D_i = (g_{1:i} - g_{2:i})^2$$

  - Colour (RGB) signals and symmetric noise:

$$D_i = (g_{R:1:i} - g_{R:2:i})^2 + (g_{G:1:i} - g_{G:2:i})^2 + (g_{B:1:i} - g_{B:2:i})^2$$

- The simplest dissimilarity for MVPs – $D_{\text{occ}} = \text{const}$
  - This constant weight corresponds to expected signal mismatches for partially occluded points which observed only in one image
  - More adequate might be a varying weight for the MVPs depending on signal mismatches for the relevant BVPs

Dissimilarity for Varying Contrast

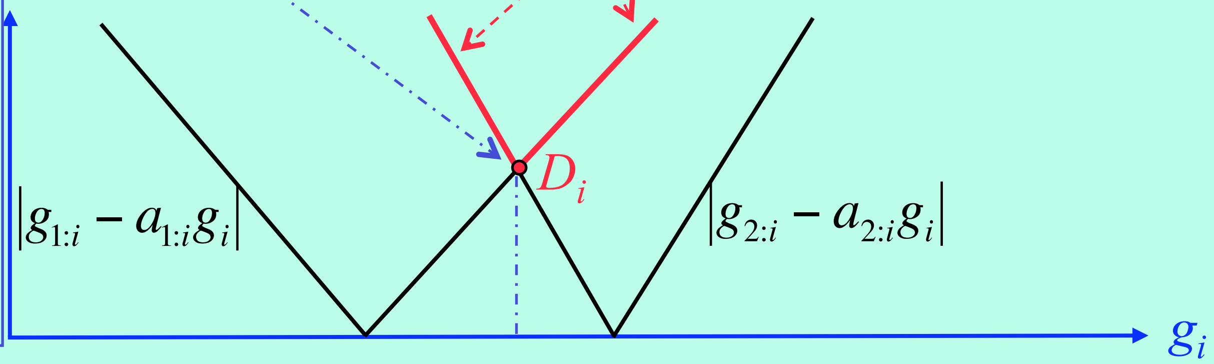
- Absolute signal dissimilarities for the BVPs ($s_i = \mathrm{B}$):

$$\left(\left(\left|g_{1:i} - a_{1:i} g_i\right|; \left|g_{2:i} - a_{2:i} g_i\right|\right) : i = 1,2,\ldots,N\right)$$

  – $(a_{1:i}\ ,\ a_{1:i} : i{=}1,\ldots,N)$ - sequences of the transfer factors

$$D_{i;a_{1:i},a_{2:i}} = \min_{g_i}\left\{\max\left\{\left|g_{1:i} - a_{1:i} g_i\right|; \left|g_{2:i} - a_{2:i} g_i\right|\right\}\right\}$$

Given these factors a, the simplest estimate of the unknown cyclopean signal minimises the maximum of the two node-wise signal dissimilarities



Minimax Parameter Estimates

Minimum by g_i condition

$$D_{i;a_{1:i},a_{2:i}} = \min_{g_i}\left\{\max\left\{|g_{1:i} - a_{1:i}g_i|;|g_{2:i} - a_{2:i}g_i|\right\}\right\} \Rightarrow \overbrace{g_{1:i} - a_{1:i}g_i = -g_{2:i} + a_{2:i}g_i}$$

$$\Rightarrow g_i = \frac{g_{1:i} + g_{2:i}}{a_{1:i} + a_{2:i}}$$

Minimax estimate of the cyclopean signal

$$\Rightarrow D_{i;\alpha} = \left|\left(g_{1:i} - \alpha\left(g_{1:i} + g_{2:i}\right)\right)\right| \equiv \left|\left(g_{2:i} - (1-\alpha)\left(g_{1:i} + g_{2:i}\right)\right)\right|$$

Relative transfer factor

where $\alpha = \dfrac{a_{1:i}}{a_{1:i} + a_{2:i}} \in [0,1]$

$$\Rightarrow \alpha \in \left[\alpha_{\min}, \alpha_{\max}\right];\ \ 0 < \alpha_{\min} \le 0.5 \le \alpha_{\max} = 1 - \alpha_{\min} < 1$$

Admissible range of distortion

Point-wise Signal Dissimilarity

$$D_i = \min_{\alpha \in [\alpha_{\min}, \alpha_{\max}]} \{D_{i;\alpha}\} = |g_{1:i} - \hat{g}_{1:i}|$$

Cyclopean signal adapted to the left signal

where $\hat{g}_{1:i} = \begin{cases} \alpha_{\min}(g_{1:i} + g_{2:i}) & \text{if } \alpha_i^\circ < \alpha_{\min} \\ g_{1:i} & \text{if } \alpha_i^\circ \in [\alpha_{\min}, \alpha_{\max}], \\ \alpha_{\max}(g_{1:i} + g_{2:i}) & \text{if } \alpha_i^\circ > \alpha_{\max} \end{cases}$

Actual relative distortion

and $\alpha_i^\circ = \dfrac{g_{1:i}}{g_{1:i} + g_{2:i}}$

The same estimates of the cyclopean signal and the relative distortion factor is valid for the squared signal dissimilarity:

$$D_{i;a_{1:i},a_{2:i}} = \min_{g_i} \left\{ \max\left\{ (g_{1:i} - a_{1:i} g_i)^2; (g_{2:i} - a_{2:i} g_i)^2 \right\} \right\} \Rightarrow D_i = (g_{1:i} - \hat{g}_{1:i})^2$$

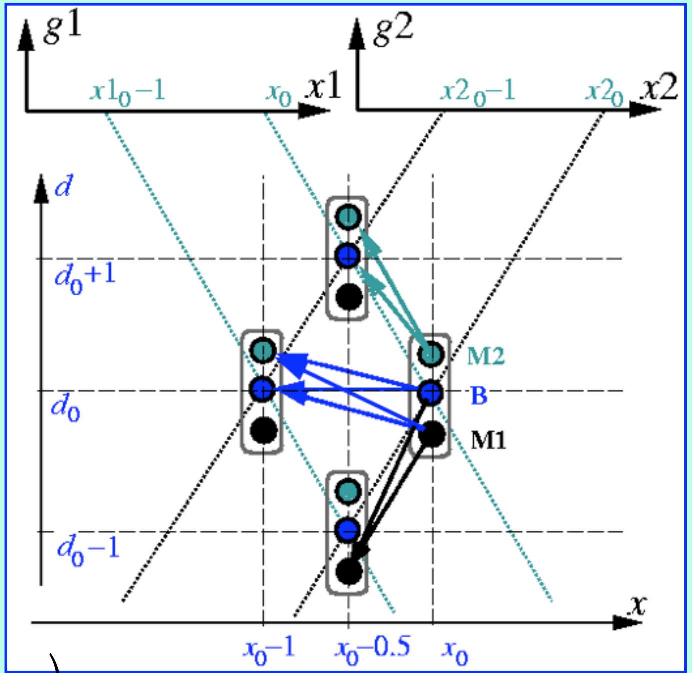
Point-wise Signal Dissimilarity

- For a BVP $(x_i, d_i, s_i = \mathrm{B})$ in a profile $\mathbf{d}$:

$$D_i \equiv D_y\left(x_i, d_i, s_i \middle| g_{1:x_i+d_i/2,y}, g_{2:x_i-d_i/2,y}\right) = \begin{cases} \alpha_{\min}\left(g_{1:i} + g_{2:i}\right) - g_{1:i} & \text{if } \dfrac{g_{1:i}}{g_{1:i} + g_{2:i}} < \alpha_{\min} \\ 0 & \text{if } \dfrac{g_{1:i}}{g_{1:i} + g_{2:i}} \in [\alpha_{\min}, \alpha_{\max}] \\ g_{1:i} - \alpha_{\max}\left(g_{1:i} + g_{2:i}\right) & \text{if } \dfrac{g_{1:i}}{g_{1:i} + g_{2:i}} > \alpha_{\max} \end{cases}$$

- For a MVP $(x_i, d_i, s_i = \mathrm{M}_1$ or $\mathrm{M}_2)$ in a profile $\mathbf{d}$: a regularising constant "dissimilarity" D_{occ} to account for partially occluded points without stereo correspondence

Total Signal Dissimilarity



- GPV-nodes along a continuous profile are subject to the visibility and ordering constraints:

$$D_y(\mathbf{d}\mid\mathbf{g}_1,\mathbf{g}_2)=\sum_{i=1}^{N} D_y\left(x_i,d_i,s_i \middle| g_{1:x_i+d_i/2,y};g_{2:x_i-d_i/2,y}\right)$$

$$\forall_{i=2,\ldots,N}\quad (x_{i-1},d_{i-1},s_{i-1})\in\mathbf{\Omega}_{x_i,d_i,s_i}$$

$$\mathbf{\Omega}_{x_i,d_i,s_i}=\begin{cases}\left\{\left(x_i-\frac{1}{2},d_i+1,\mathrm{B}\right),\left(x_i-\frac{1}{2},d_i+1,\mathrm{M}_2\right)\right\} & \text{if}\quad s_i=\mathrm{M}_2\\ \left\{\left(x_i-\frac{1}{2},d_i-1,\mathrm{M}_1\right),\left(x_i-1,d_i,\mathrm{B}\right),\left(x_i-1,d_i,\mathrm{M}_2\right)\right\} & \text{if}\quad s_i=\mathrm{B}\\ \left\{\left(x_i-\frac{1}{2},d_i-1,\mathrm{M}_1\right),\left(x_{i-1}-1,d_i,\mathrm{B}\right)\right\} & \text{if}\quad s_i=\mathrm{M}_1\end{cases}$$

Allowable preceding GPV-nodes along every profile

Dynamic Programming (DP)

- DP exhausts all the profiles under the constraints Ω and find the global minimum of the total dissimilarity
  - The search: by a successive pass along the x-axis of a GPV
  - At any current location, x_i, all possible GPV-nodes (x_i,d_i,s_i) are examined in order to calculate and store the current **total** potentially optimal dissimilarity $D_{\text{total}}(x_i,d_i,s_i)$
    - Potential optimality: the stored dissimilarity is optimal provided that this node will belong to the globally optimal solution
  - For every node (x_i,d_i,s_i), the potentially optimal transition to one of the preceding nodes $(x_{i-1},d_{i-1},s_{i-1})$ in Ω is stored

Basic DP Computation

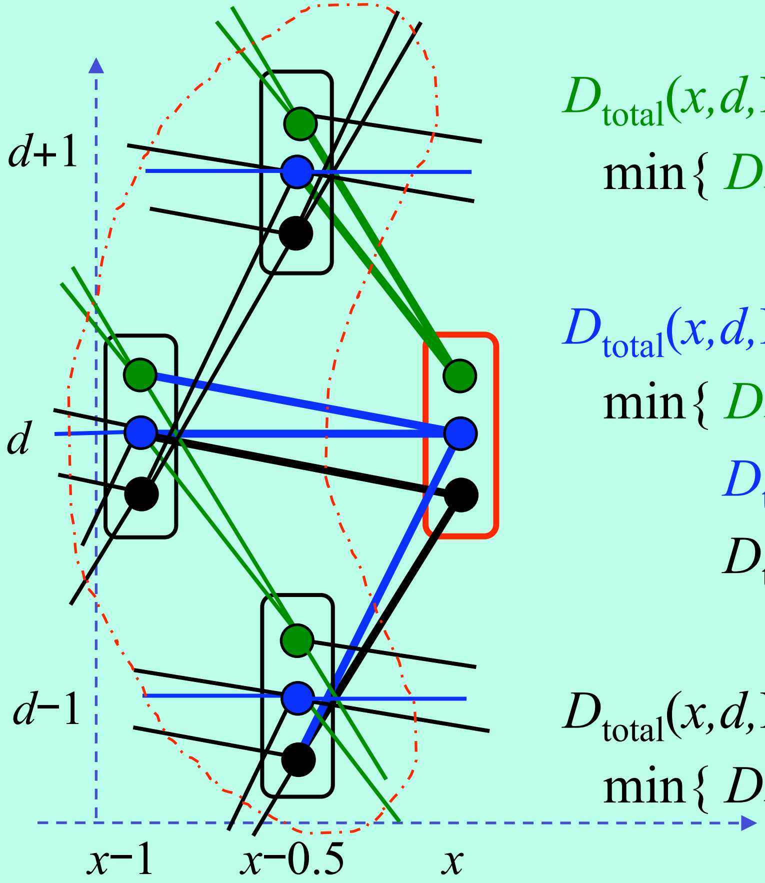
– $D_{\text{total}}(x_i,d_i,s_i)$ - the **total minimal signal dissimilarity** for the potentially optimal backward path from the node (x_i,d_i,s_i)

- **Recurrent DP computation**:

$$\begin{cases} D_{\text{total}}\left(x_i,d_i,s_i\right) = D_y\left(x_i,d_i,s_i \middle| g_{1:x_i+d_i/2,y}; g_{2:x_i-d_i/2,y}\right) + D_{\text{total}}\left(x^*_{i-1},d^*_{i-1},s^*_{i-1}\right) \\ T_{\text{back}}\left(x_i,d_i,s_i\right) \equiv \left(x^*_{i-1},d^*_{i-1},s^*_{i-1}\right) = \underset{\left(x_{i-1},d_{i-1},s_{i-1}\right)\in\Omega_{x_i,d_i,s_i}}{\arg\ \min} \left\{D_{\text{total}}\left(x_{i-1},d_{i-1},s_{i-1}\right)\right\} \end{cases}$$

where $T_{\text{back}}(x_i,d_i,s_i)$ is an indicator function of the potentially optimum backward transition from the node (x_i,d_i,s_i)

Basic DP Computation

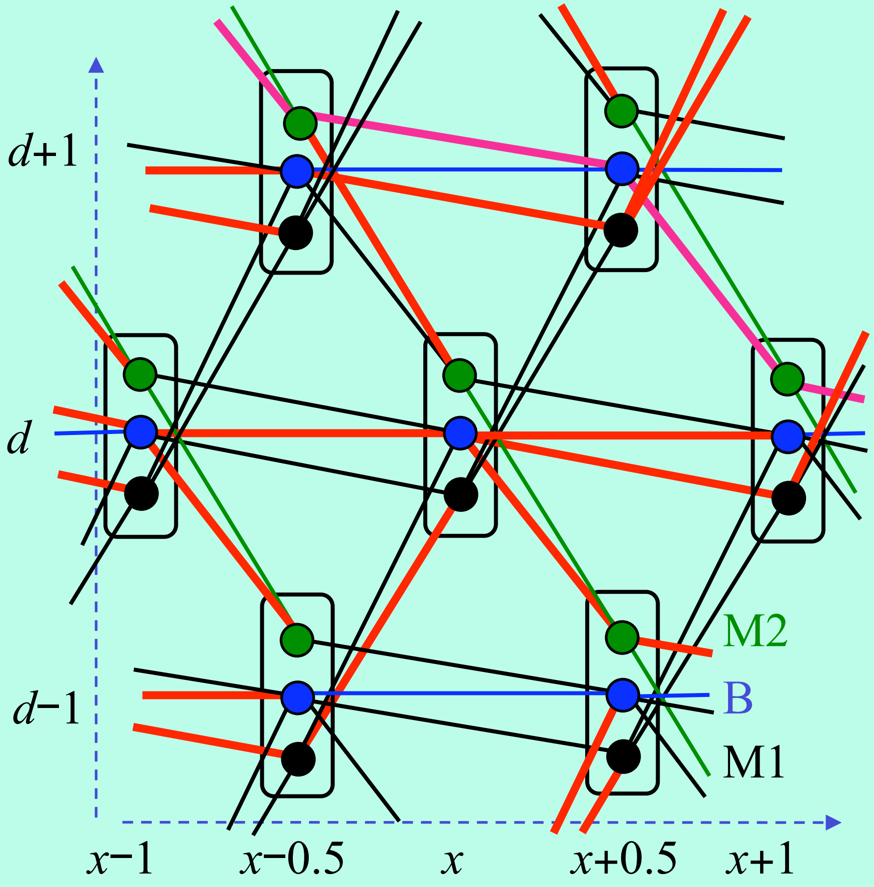


$D_{total}(x,d,\mathrm{M2}) = D_{occ} + \min\{\, D_{total}(x-0.5,d+1,\mathrm{M2});\ D_{total}(x-0.5,d+1,\mathrm{B}) \,\}$

$D_{total}(x,d,\mathrm{B}) = \min\{\, D_{total}(x-1,d,\mathrm{M2}) + D_y(x,d;x_{prB:M2},d_{prB:M2});$
$\quad D_{total}(x-1,d+1,\mathrm{B}) + D_y(x,d;x-1,d);$
$\quad D_{total}(x-0.5,d-1,\mathrm{M1}) + D_y(x,d;x_{prB:M1},d_{prB:M1}) \,\}$

$D_{total}(x,d,\mathrm{M1}) = D_{occ} + \min\{\, D_{total}(x-0.5,d-1,\mathrm{M1});\ D_{total}(x-1,d,\mathrm{B}) \,\}$

Basic DP Computation



- $T_{\text{back}}(x_i,d_i,s_i)$ - the potentially optimal backward transition from every GPV-node
- The optimal profile is formed from a backward sequence of potentially optimal transitions
  - The sequence starts (i.e. the profile ends) at a rightmost node ($x_N = x_{\max}$ or $x_{\max}-0.5$) such that its total dissimilarity $D_{\text{total}}(x_N,d_N,s_N)$ is minimal

Basic DP Computation

– The goal optimal profile minimises the total signal dissimilarity

- After passing a range $[x_{\min}, x_{\max}]$ of the x-coordinates, the optimal profile is recovered backward by passing via the stored potentially optimal backward transitions:

$$\begin{cases} D_y(\mathbf{d} \mid g_1, g_2) = S\left(x_N^*, d_N^*, s_N^*\right) \text{ where } \left(x_N^*, d_N^*, s_N^*\right) = \underset{\substack{x_N \in \left\{x_{\max} - \frac{1}{2}, x_{\max}\right\} \\ d_N \in \left[d_{\min}, d_{\max}\right]; \\ s_N \in \{\text{M1}, \text{B}, \text{M2}\}}}{\arg\min} D_{\text{total}}\left(x_N, d_N, s_N\right) \\ \left(x_{i-1}^*, d_{i-1}^*, s_{i-1}^*\right) = T_{\text{back}}\left(x_i^*, d_i^*, s_i^*\right); \quad i = N, N-1, \ldots, 2 \end{cases}$$

Interdependent Transfer Factors

Floating range of the factors: for their spatial dependence

i'_{B} - the index of the BVP preceding the BVP i along the profile

$$\varepsilon \in [\varepsilon_{\min}, \varepsilon_{\max}]; \;\; 0 < \varepsilon_{\min} \le 0.5 \le \varepsilon_{\max} = 1 - \varepsilon_{\min} < 1$$

"Difference" factor

$$\hat{g}_{1:i}^{\min} = \min_{\varepsilon} \left| \hat{g}_{1:i'_{\mathrm{B}}} + \varepsilon \left(g_{1:i} + g_{2:i} - g_{1:i'_{\mathrm{B}}} - g_{2:i'_{\mathrm{B}}} \right) \right|$$

$$\hat{g}_{1:i}^{\max} = \max_{\varepsilon} \left| \hat{g}_{1:i'_{\mathrm{B}}} + \varepsilon \left(g_{1:i} + g_{2:i} - g_{1:i'_{\mathrm{B}}} - g_{2:i'_{\mathrm{B}}} \right) \right|$$

Floating range of signal adaptation depending on potentially optimal paths

$$D_i = \left| g_{1:i} - \hat{g}_{1:i} \right|; \quad \hat{g}_{1:i} = \begin{cases} \hat{g}_{1:i}^{\min} & \text{if } g_{1:i} < \hat{g}_{1:i}^{\min} \\ g_{1:i} & \text{if } g_{1:i} \in [\hat{g}_{1:i}^{\min}; \hat{g}_{1:i}^{\max}] \\ \hat{g}_{1:i}^{\max} & \text{if } g_{1:i} > \hat{g}_{1:i}^{\min} \end{cases}$$

Point-wise signal difference for the approximate DP stereo

Symmetric DP Stereo