Dynamic Programming Stereo

COMPSCI 773 S1 T

VISION GUIDED CONTROL

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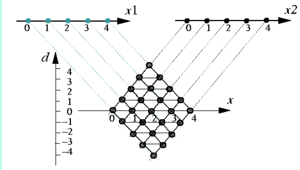
Markov Chain Model of a Profile

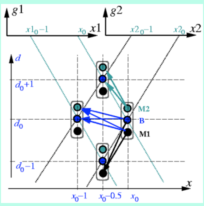
- Accounts for symmetry of stereo channels, visibility of 3-D points and discontinuities due to occlusions
- Single continuous surface only - the **ordering** constraint



Graph of profile variants (GPV)

Admissible Transitions

- Partial occlusions impose the **visibility constraint** on x-disparities in the symmetric coordinates (x,d):



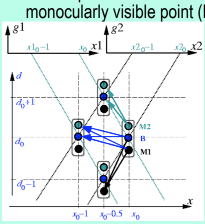
$$d_{x,y} - 1 \le d_{x-0.5,y} \le d_{x,y} + 1$$
$$d_{x,y} - 2 \le d_{x-1,y} \le d_{x,y} + 2$$

Assumption: every 3-D point is visible either binocularly (B) or monocularly by the left (M1) or right (M2) camera

Transitions between the nodes in a GPV form admissible variants of an epipolar profile y

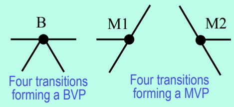
Admissible Transitions

Depending on the incoming and outcoming transition, the GPV node represents either a binocularly visible point (BVP), or a monocularly visible point (MVP) occluded in one image

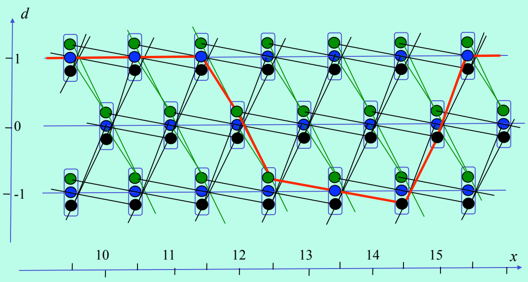


Each node $\mathbf{v} = (x,d,s)$ has three visibility states s indicating the binocular, s = B, or only monocular, s = M1 or M2, viewing

Only eight transitions are allowed in a GPV:



Admissible Transitions



Admissible Transitions

BVP :

$(x,d,\mathrm{B}) \Rightarrow (g_{1:x+d/2}; g_{2:x-d/2});$

$(g_{1:x_{pr}+d_{pr}/2}; g_{2:x_{pr}-d_{pr}/2})$

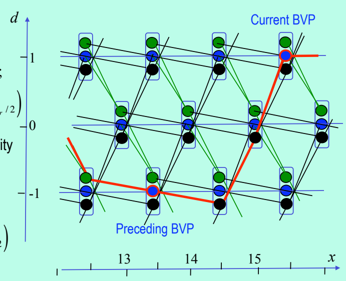
Pixel-wise signal dissimilarity

MVP :

$(x,d,\mathrm{ML}); (x,d,\mathrm{MR})$

$\Rightarrow (g_{1:x_{pr}+d_{pr}/2}; g_{2:x_{pr}-d_{pr}/2})$

Fixed "dissimilarity" weight



Markov Chain x-Profile Model

- Probability of a profile $\mathbf{d} = [(x_i, d_i, s_i): i = 1,\ldots,n]$:

$$\Pr(\mathbf{d}|\mathbf{g}_L,\mathbf{g}_R) = p(x_1,d_1,s_1|\mathbf{g}_1,\mathbf{g}_2)\prod_{i=2}^{n} p(x_i,d_i,s_i|x_{i-1},d_{i-1},s_{i-1};\mathbf{g}_1,\mathbf{g}_2)$$

  where each term depends on transitions from the GVP-node in state s' to the node in state s'' along the profile
  - Transitions are limited by the visibility states along a GVP
  - Probability $p(x_i,d_i,\mathrm{B}|x_{i-1},d_{i-1},s_{i-1};\mathbf{g}_1,\mathbf{g}_2)$ of a transition to state B depends on dissimilarity between the corresponding image signals for the current BVP on a profile's variant
    - It can also depend on the signals for the preceding BVP along this variant
  - Transition probabilities to the MVPs can be related to those for the BVPs
    - Typical simplification: a constant dissimilarity weight

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Corresponding Signals Models

- Symmetric photometric model: signal distortions in the stereo images g_1 and g_2 w.r.t. the unobserved "ideal" cyclopean image (or ortho-image) g of a 3-D scene
- The simplest model: $g_{1:x_1,y_1} = g_{x,y} + n_{1:x,y}; \quad g_{2:x_2,y_2} = g_{x,y} + n_{2:x,y}$

  Independent central-symmetric noise (monotone decrease of probability density)

  Distortion-less signal for a 3-D point
- Model of contrast distortions - positive transfer factors, a, varying over the field of view of each camera and pixel-wise independent random noise, n, of image sensors:

$$g_{1:x_1,y_1} = a_{1:x,y}g_{x,y} + n_{1:x,y}; \quad g_{2:x_2,y_2} = a_{2:x,y}g_{x,y} + n_{2:x,y}$$

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Model of Contrast Distortions

- Transfer factors: strong dependences for adjacent BVPs to account for visual resemblance of corresponding areas
- **Symmetric difference model** of the interdependence:
  - Limited direct proportion between the noiseless increments within a range: $0 < e_{\min} \le e_{\max}$ of difference factors e:

$$\min_{e_1\in[e_{\min},e_{\max}]}\left\{e_1\left(g_{x,y}-g_{x',y'}\right)\right\} \le a_{1:x,y}g_{x,y} - a_{1:x',y'}g_{x',y'} \le \max_{e_1\in[e_{\min},e_{\max}]}\left\{e_1\left(g_{x,y}-g_{x',y'}\right)\right\}$$

$$\min_{e_2\in[e_{\min},e_{\max}]}\left\{e_2\left(g_{x,y}-g_{x',y'}\right)\right\} \le a_{2:x,y}g_{x,y} - a_{2:x',y'}g_{x',y'} \le \max_{e_2\in[e_{\min},e_{\max}]}\left\{e_2\left(g_{x,y}-g_{x',y'}\right)\right\}$$

Neighbouring BVPs along the same epipolar profile

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Model of Contrast Distortions

- Assuming that both the ortho-image signals g and random noise n are statistically independent, the difference model results in a Markov chain of signals for the successive BVPs and in the independent signals for the MVPs along a profile
  - Under a known 3-D profile, statistical estimates for the ortho-image g and transfer factors a_1, a_2 are derived on the basis of assumptions about the random noise and allowable spatial variations of the factors
- Theoretically justified part of a similarity measure is based on dissimilarity between the corresponding signals for the BVPs
- Heuristic regularising part of the measure relates to the MVPs
  - E.g. the assumptions of the links between the MVPs and near BVPs

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Dynamic Programming Stereo

- Simplified notation: the y-coordinate is omitted
  - $\mathbf{d} = ((x_i, d_i, s_i) : i = 1,2,\ldots,N)$ - a digital profile as a sequence of the adjacent nodes in a GPV along allowable transitions

    $\mathbf{g} = (g_i: i=1,2,\ldots,N)$; $\mathbf{g}_1 = (g_{1:i} : i=1,2,\ldots,N_1)$; $\mathbf{g}_2 = (g_{2:i} : i=1,2,\ldots,N_2)$ - sequences of the cyclopean, left, and right image signals for the profile $\mathbf{d}$
  - For brevity: $g_i \equiv g_{x_i,y}; \quad g_{1:i} \equiv g_{1:x_i+d_i/2,y}; \quad g_{2:i} \equiv g_{2:x_i-d_i/2,y}$
- Signal model with only noise: $g_{1:i} = g_i + n_{1:i}; \quad g_{2:i} = g_i + n_{2:i}$
- Signal model with varying contrast:

$$g_{1:i} = a_{1:i}g_i + n_{1:i}; \quad g_{2:i} = a_{2:i}g_i + n_{2:i}$$

- Signal model with varying contrast and offset:

$$\Delta g_{1:i,i-1} = e_{1:i}\Delta g_{i,i-1} + \Delta n_{1:i,i-1}; \quad \Delta g_{2:i,i-1} = e_{2:i}\Delta g_{i,i-1} + \Delta n_{2:i,i-1}$$

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Pixel-wise Signal Dissimilarity

- The simplest symmetric signal model (only noise):
  - Grayscale signals and symmetric noise: $g_{1:i} = g_i + n_{1:i}; \quad g_{2:i} = g_i + n_{2:i}$

$$\min_{g_i}\left\{\max\left\{(g_{1:i}-g_i)^2, (g_{1:i}-g_i)^2\right\}\right\} \Rightarrow g_i = \frac{g_{1:i}+g_{2:i}}{2} \Rightarrow D_i = (g_{1:i}-g_{2:i})^2$$

  - Colour (RGB) signals and symmetric noise:

$$D_i = (g_{R:1:i}-g_{R:2:i})^2 + (g_{G:1:i}-g_{G:2:i})^2 + (g_{B:1:i}-g_{B:2:i})^2$$

- The simplest dissimilarity for MVPs – $D_{\mathrm{occ}} = \mathrm{const}$
  - This constant weight corresponds to expected signal mismatches for partially occluded points which observed only in one image
  - More adequate might be a varying weight for the MVPs depending on signal mismatches for the relevant BVPs

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Dissimilarity for Varying Contrast

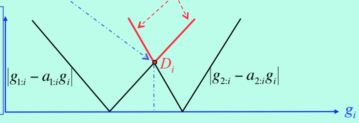
- Absolute signal dissimilarities for the BVPs ($s_i = \mathrm{B}$):

$$\left(\left(|g_{1:i} - a_{1:i}g_i|;|g_{2:i} - a_{2:i}g_i|\right) : i = 1,2,\dots,N\right)$$

  – $(a_{1:i}\ ,\ a_{1:i}\ :\ i{=}1,\dots,N)$ - sequences of the transfer factors

$$D_{i;a_{1:i},a_{2:i}} = \min_{g_i}\left\{\max\left\{|g_{1:i} - a_{1:i}g_i|;|g_{2:i} - a_{2:i}g_i|\right\}\right\}$$

Given these factors a, the simplest estimate of the unknown cyclopean signal minimises the maximum of the two node-wise signal dissimilarities



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Minimax Parameter Estimates

$$D_{i;a_{1:i},a_{2:i}} = \min_{g_i}\left\{\max\left\{|g_{1:i} - a_{1:i}g_i|;|g_{2:i} - a_{2:i}g_i|\right\}\right\} \Rightarrow \overbrace{g_{1:i} - a_{1:i}g_i = -g_{2:i} + a_{2:i}g_i}$$

Minimum by g_i condition

$$\Rightarrow g_i = \frac{g_{1:i} + g_{2:i}}{a_{1:i} + a_{2:i}}$$

Minimax estimate of the cyclopean signal

$$\Rightarrow D_{i;\alpha} = \left|\left(g_{1:i} - \alpha(g_{1:i} + g_{2:i})\right)\right| \equiv \left|\left(g_{2:i} - (1-\alpha)(g_{1:i} + g_{2:i})\right)\right|$$

Relative transfer factor

where $\alpha = \dfrac{a_{1:i}}{a_{1:i} + a_{2:i}} \in [0,1]$

$$\Rightarrow \alpha \in [\alpha_{\min}, \alpha_{\max}];\ \ 0 < \alpha_{\min} \le 0.5 \le \alpha_{\max} = 1 - \alpha_{\min} < 1$$

Admissible range of distortion

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Point-wise Signal Dissimilarity

$$D_i = \min_{\alpha \in [\alpha_{\min},\alpha_{\max}]}\{D_{i;\alpha}\} = |g_{1:i} - \hat{g}_{1:i}|$$

Cyclopean signal adapted to the left signal

$$\text{where } \hat{g}_{1:i} = \begin{cases} \alpha_{\min}(g_{1:i} + g_{2:i}) & \text{if } \alpha_i^\circ < \alpha_{\min} \\ g_{1:i} & \text{if } \alpha_i^\circ \in [\alpha_{\min}, \alpha_{\max}], \\ \alpha_{\max}(g_{1:i} + g_{2:i}) & \text{if } \alpha_i^\circ > \alpha_{\max} \end{cases}$$

Actual relative distortion

$$\text{and } \alpha_i^\circ = \frac{g_{1:i}}{g_{1:i} + g_{2:i}}$$

The same estimates of the cyclopean signal and the relative distortion factor is valid for the squared signal dissimilarity:

$$D_{i;a_{1:i},a_{2:i}} = \min_{g_i}\left\{\max\left\{(g_{1:i} - a_{1:i}g_i)^2;(g_{2:i} - a_{2:i}g_i)^2\right\}\right\} \Rightarrow D_i = (g_{1:i} - \hat{g}_{1:i})^2$$

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Point-wise Signal Dissimilarity

- For a BVP $(x_i,d_i,s_i{=}\mathrm{B})$ in a profile $\mathbf{d}$:

$$D_i \equiv D_y\left(x_i,d_i,s_i \middle| g_{1:x_i+d_i/2,y}, g_{2:x_i-d_i/2,y}\right) = \begin{cases} \alpha_{\min}(g_{1:i} + g_{2:i}) - g_{1:i} & \text{if } \dfrac{g_{1:i}}{g_{1:i} + g_{2:i}} < \alpha_{\min} \\ 0 & \text{if } \dfrac{g_{1:i}}{g_{1:i} + g_{2:i}} \in [\alpha_{\min}, \alpha_{\max}] \\ g_{1:i} - \alpha_{\max}(g_{1:i} + g_{2:i}) & \text{if } \dfrac{g_{1:i}}{g_{1:i} + g_{2:i}} > \alpha_{\max} \end{cases}$$

- For a MVP $(x_i,d_i,s_i{=}\mathrm{M}_1$ or $\mathrm{M}_2)$ in a profile $\mathbf{d}$: a regularising constant "dissimilarity" D_{occ} to account for partially occluded points without stereo correspondence

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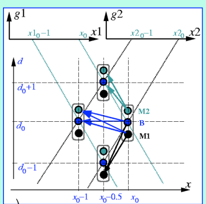
Total Signal Dissimilarity



- GPV-nodes along a continuous profile are subject to the visibility and ordering constraints:

$$D_y(\mathbf{d}\,|\,\mathbf{g}_1,\mathbf{g}_2) = \sum_{i=1}^{N} D_y\left(x_i,d_i,s_i \middle| g_{1:x_i+d_i/2,y}; g_{2:x_i-d_i/2,y}\right)$$

$$\forall_{i=2,\dots,N}\quad (x_{i-1},d_{i-1},s_{i-1}) \in \mathbf{\Omega}_{x_i,d_i,s_i}$$

$$\mathbf{\Omega}_{x_i,d_i,s_i} = \begin{cases} \left\{\left(x_i - \tfrac{1}{2}, d_i + 1, \mathrm{B}\right), \left(x_i - \tfrac{1}{2}, d_i + 1, \mathrm{M}_2\right)\right\} & \text{if } s_i = \mathrm{M}_2 \\ \left\{\left(x_i - \tfrac{1}{2}, d_i - 1, \mathrm{M}_1\right), \left(x_i - 1, d_i, \mathrm{B}\right), \left(x_i - 1, d_i, \mathrm{M}_2\right)\right\} & \text{if } s_i = \mathrm{B} \\ \left\{\left(x_i - \tfrac{1}{2}, d_i - 1, \mathrm{M}_1\right), \left(x_{i-1} - 1, d_i, \mathrm{B}\right)\right\} & \text{if } s_i = \mathrm{M}_1 \end{cases}$$

Allowable preceding GPV-nodes along every profile

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Dynamic Programming (DP)

- DP exhausts all the profiles under the constraints Ω and find the global minimum of the total dissimilarity
  – The search: by a successive pass along the x-axis of a GPV
  – At any current location, x_i, all possible GPV-nodes (x_i,d_i,s_i) are examined in order to calculate and store the current **total** potentially optimal dissimilarity $D_{\mathrm{total}}(x_i,d_i,s_i)$
    - Potential optimality: the stored dissimilarity is optimal provided that this node will belong to the globally optimal solution
  – For every node (x_i,d_i,s_i), the potentially optimal transition to one of the preceding nodes $(x_{i-1},d_{i-1},s_{i-1})$ in Ω is stored

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Basic DP Computation

– $D_{total}(x_i,d_i,s_i)$ - the **total minimal signal dissimilarity** for the potentially optimal backward path from the node (x_i,d_i,s_i)

- **Recurrent DP computation**:

$$\begin{cases} D_{total}(x_i,d_i,s_i) = D_y\left(x_i,d_i,s_i \middle| g_{1:x_i+d_i/2,y}; g_{2:x_i-d_i/2,y}\right) + D_{total}\left(x^*_{i-1},d^*_{i-1},s^*_{i-1}\right) \\ T_{back}(x_i,d_i,s_i) \equiv \left(x^*_{i-1},d^*_{i-1},s^*_{i-1}\right) = \underset{(x_{i-1},d_{i-1},s_{i-1})\in\Omega_{x_i,d_i,s_i}}{\arg\min} \left\{D_{total}\left(x_{i-1},d_{i-1},s_{i-1}\right)\right\} \end{cases}$$

where $T_{back}(x_i,d_i,s_i)$ is an indicator function of the potentially optimum backward transition from the node (x_i,d_i,s_i)

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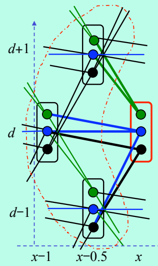
Basic DP Computation



$D_{total}(x,d,\mathrm{M2}) = D_{occ} +$
$\min\{ D_{total}(x-0.5,d+1,\mathrm{M2}); D_{total}(x-0.5,d+1,\mathrm{B}) \}$

$D_{total}(x,d,\mathrm{B}) =$
$\min\{ D_{total}(x-1,d,\mathrm{M2}) + D_y(x,d;x_{prB:M2},d_{prB:M2});$
$D_{total}(x-1,d+1,\mathrm{B}) + D_y(x,d;x-1,d);$
$D_{total}(x-0.5,d-1,\mathrm{M1}) + D_y(x,d;x_{prB:M1},d_{prB:M1}) \}$

$D_{total}(x,d,\mathrm{M1}) = D_{occ} +$
$\min\{ D_{total}(x-0.5,d-1,\mathrm{M1}); D_{total}(x-1,d,\mathrm{B}) \}$

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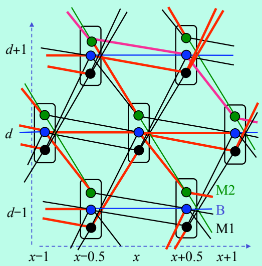
Basic DP Computation



- $T_{back}(x_i,d_i,s_i)$ - the potentially optimal backward transition from every GPV-node
- The optimal profile is formed from a backward sequence of potentially optimal transitions
  – The sequence starts (i.e. the profile ends) at a rightmost node ($x_N = x_{max}$ or $x_{max}-0.5$) such that its total dissimilarity $D_{total}(x_N,d_N,s_N)$ is minimal

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Basic DP Computation

– The goal optimal profile minimises the total signal dissimilarity

- After passing a range $[x_{min}, x_{max}]$ of the x-coordinates, the optimal profile is recovered backward by passing via the stored potentially optimal backward transitions:

$$\begin{cases} D_y(\mathbf{d}\mid g_1,g_2) = S\left(x^*_N,d^*_N,s^*_N\right) \text{ where } \left(x^*_N,d^*_N,s^*_N\right) = \underset{\substack{x_N\in\left\{x_{max}-\frac{1}{2},x_{max}\right\} \\ d_N\in\left[d_{min},d_{max}\right]; \\ s_N\in\{\mathrm{M1,B,M2}\}}}{\arg\min} D_{total}\left(x_N,d_N,s_N\right) \\ \left(x^*_{i-1},d^*_{i-1},s^*_{i-1}\right) = T_{back}\left(x^*_i,d^*_i,s^*_i\right); \quad i = N,N-1,\ldots,2 \end{cases}$$

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Interdependent Transfer Factors

Floating range of the factors: for their spatial dependence

i'_B - the index of the BVP preceding the BVP i along the profile

$\varepsilon\in\left[\varepsilon_{min},\varepsilon_{max}\right]; \ 0<\varepsilon_{min}\le 0.5\le\varepsilon_{max}=1-\varepsilon_{min}<1$ "Difference" factor

$$\left.\begin{aligned} \hat{g}^{min}_{1:i} &= \min_{\varepsilon}\left|\hat{g}_{1:i'_B} + \varepsilon\left(g_{1:i}+g_{2:i}-g_{1:i'_B}-g_{2:i'_B}\right)\right| \\ \hat{g}^{max}_{1:i} &= \max_{\varepsilon}\left|\hat{g}_{1:i'_B} + \varepsilon\left(g_{1:i}+g_{2:i}-g_{1:i'_B}-g_{2:i'_B}\right)\right| \end{aligned}\right\}$$

Floating range of signal adaptation depending on potentially optimal paths

$$D_i = \left|g_{1:i}-\hat{g}_{1:i}\right|; \quad \hat{g}_{1:i} = \begin{cases} \hat{g}^{min}_{1:i} & \text{if } g_{1:i} < \hat{g}^{min}_{1:i} \\ g_{1:i} & \text{if } g_{1:i}\in[\hat{g}^{min}_{1:i};\hat{g}^{max}_{1:i}] \\ \hat{g}^{max}_{1:i} & \text{if } g_{1:i} > \hat{g}^{min}_{1:i} \end{cases}$$

Point-wise signal difference for the approximate DP stereo

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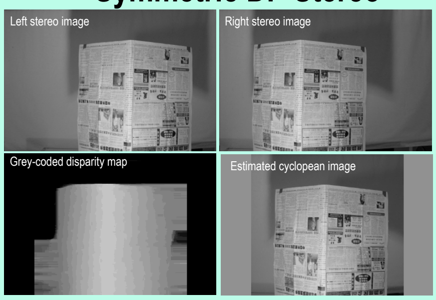
Symmetric DP Stereo



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