Correlation Based Matching

COMPSCI 773 S1 T

VISION GUIDED CONTROL

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Local and Global Optimisation

- 3-D reconstruction:
  - Search for the **max similarity** (min dissimilarity) between the corresponding regions or pixels in images of a stereo pair
  - **Basic problem**: an adequate measure for similarity (dissimilarity)
- Similarity (or dissimilarity) measure has to account for real image distortions due to separate image acquisition and different views:
  - Global and local uniform or non-uniform contrast / offset differences between corresponding signals
  - Geometric differences (projective distortions, partial occlusions)
- Similarity (or dissimilarity) has to include regularising constraints:
  - To deal with **partial occlusions** or **multiple equivalent** optima
  - For a *single continuous surface*: **visibility** and **ordering constraints**

Note: scene differences due to acquisition at different time moments; occluded walls of high buildings; different brightness / contrast

Local and Global Optimisation

- Possible scenarios of reconstructing a 3-D scene:
  1. Exhausting Δ^{NM} variants (Δ - range of disparities) of 3D surfaces in the space XYD of stereo disparities by **global constrained optimisation**

$$\mathbf{d}^* = \max_{\mathbf{d}} \left\{ \text{Similarity}(\mathbf{d} \mid \mathbf{g}_L, \mathbf{g}_R) \right\} \text{ or } \mathbf{d}^* = \min_{\mathbf{d}} \left\{ \text{Dissimilarity}(\mathbf{d} \mid \mathbf{g}_L, \mathbf{g}_R) \right\}$$

     - Constraints on neighbouring disparities (to ensure smoothness and visibility of 3D surfaces)
  2. Independent selection of each 3-D point by **local optimisation**
     - May produce physically inconsistent surfaces (violating visibility constraints)
  3. Successive search by **local optimisation** for each next small surface patch in order to add it to the already found part of the goal surface
     - Typically, leads to ***accumulation of local errors*** and thus to arbitrarily large errors in a reconstructed surface

Local Optimisation

- **Pros**:
  - Usually, quite simple computations
  - Easily accounts for both x- and y-disparities in the images
- **Cons** for independent selection of 3-D points:
  - Surfaces found may violate visibility and continuity constraints
- **Cons** for guiding each next search by the current surface:
  - Due to accumulation of local errors, search regions after a few steps may become completely wrong with no cues how to return to a true surface
- **Cons** in both cases:
  - Needs intensive on- or off-line editing of any reconstructed 3-D scene to fix inevitable large errors

Simplifying Assumptions

- Elements to match by local correlation – relatively small image windows of fixed or (less frequently) adaptable size
- Pixel correspondence is given by the window position such that the similarity score within a search region is maximised
- Assumptions:
  - Canonical stereo geometry with parallel optical axes (no y-disparity)
  - Frontal (or slant) planar surfaces, at least, approximately
  - Simple models of signal deviations between the images:
    - Only uniform independent Gaussian or any central-symmetric noise
    - Uniform contrast and offset distortions in addition to such a noise

Correspondence by Correlation

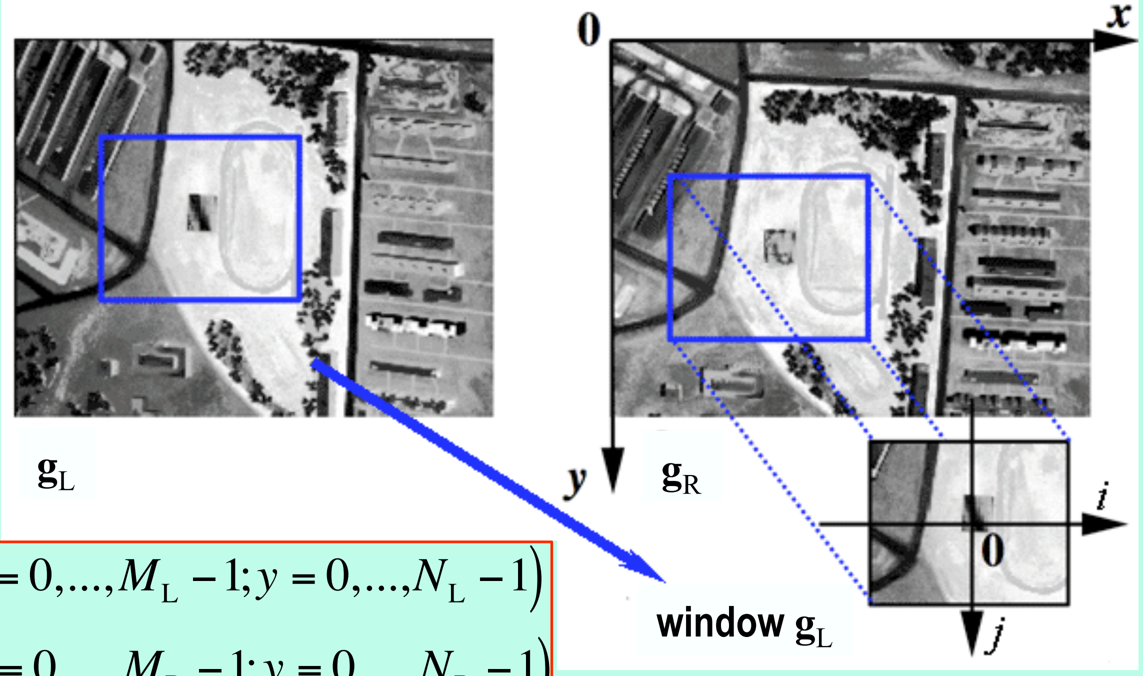
Matching rectangular windows of size $(2a+1)\times(2b+1)$ representing a frontal planar surface

Window in $\mathbf{g}_L$:

$$\left(g_{L:x'+i,y'+j} : i = -a,\ldots,0,\ldots,a; \; j = -b,\ldots,0,\ldots,b \right)$$

Window in $\mathbf{g}_R$:

$$\left(g_{R:x+i,y+j} : i = -a,\ldots,0,\ldots,a; \; j = -b,\ldots,0,\ldots,b \right)$$

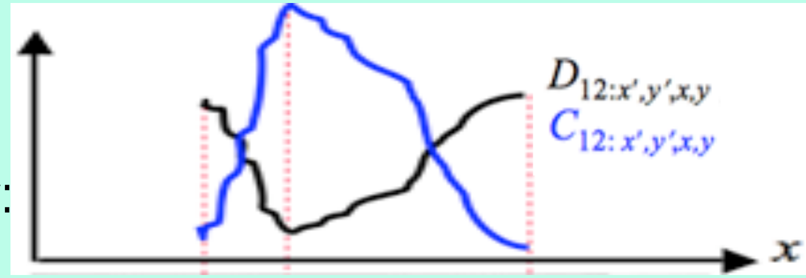


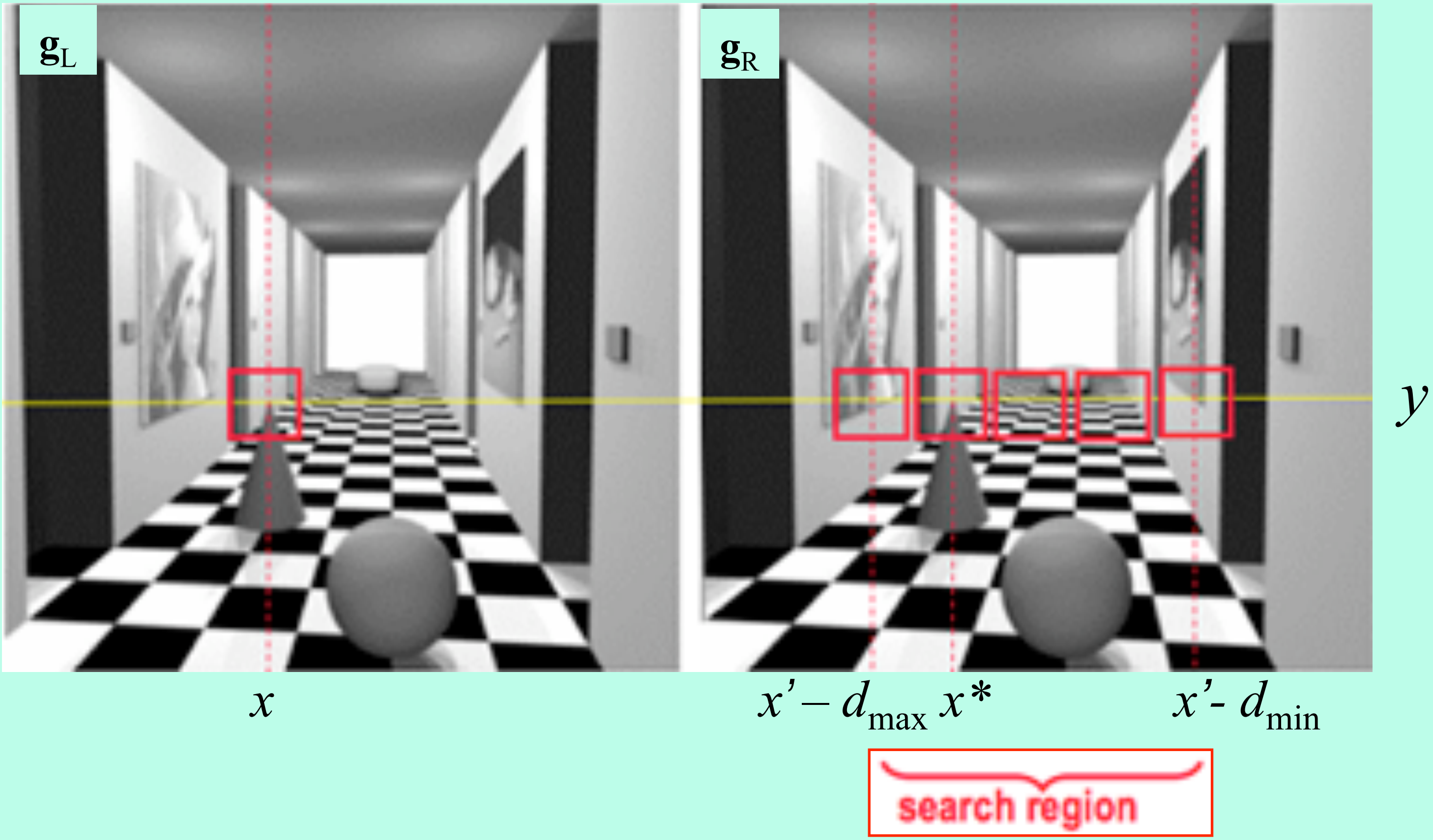
$$\mathbf{g}_L = \left(g_{L:x,y} : x = 0,\ldots,M_L - 1; y = 0,\ldots,N_L - 1 \right)$$

$$\mathbf{g}_R = \left(g_{R:x,y} : x = 0,\ldots,M_R - 1; y = 0,\ldots,N_R - 1 \right)$$

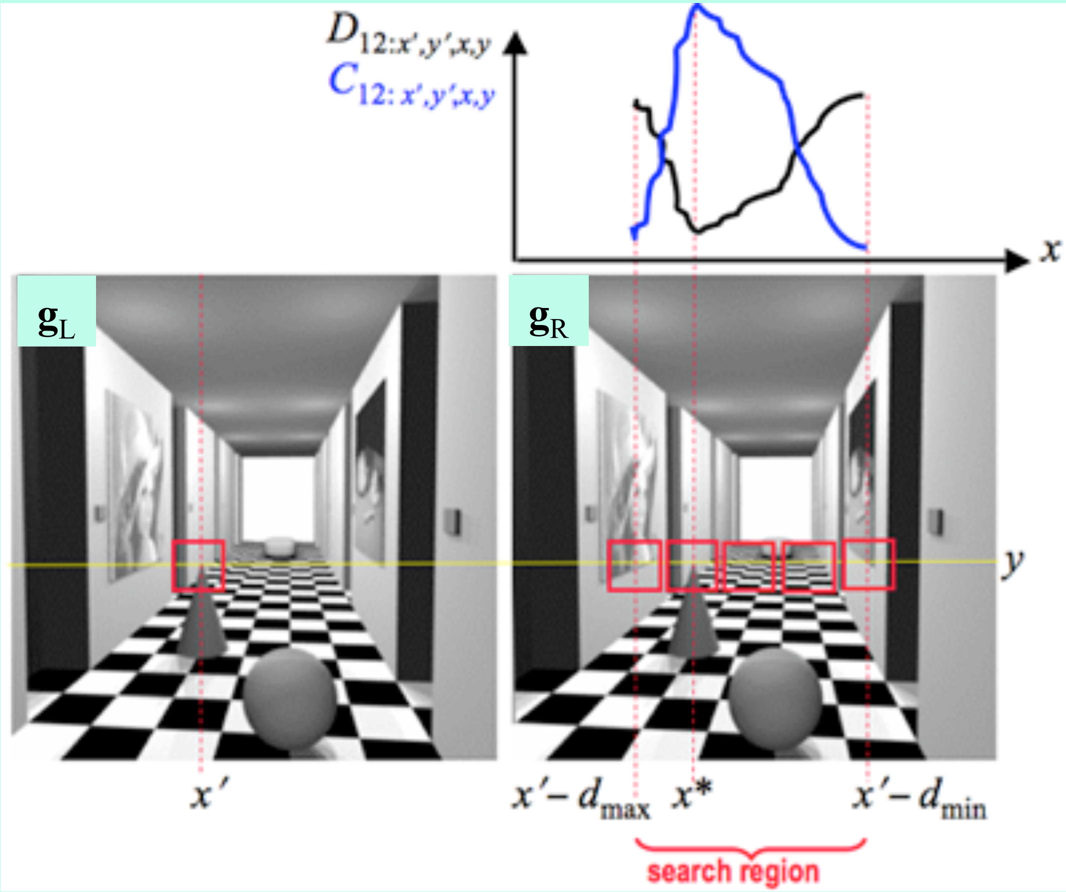
Correspondence by Correlation

Search region for the canonical geometry: along the conjugate scan-lines y





Correspondence by Correlation



For any point x' in g_L, the corresponding point (x^*,y) in g_R minimises the Euclidean distance:

$$x^* = \arg\min_{x \in SR} D_{LR:x',y;x,y}$$

or maximises the correlation:

$$x^* = \arg\max_{x \in SR} C_{LR:x',y;x,y}$$

between the fixed window in g_L and the candidate window being moved across g_R

Similarity (Dissimilarity) Score

- The **similarity** or **dissimilarity score**: by minimising the distance between the corresponding signals in the windows <u>with respect to allowable relative distortion</u>
- Minimum squared distance ⇔ the maximal probability of signal matching for the uniform Gaussian noise
  - Actually, any <u>central-symmetric</u> noise such that its probability distribution monotonously decreases with the squared noise
- Minimum absolute distance if the probability distribution monotonously decreases with the absolute noise

Symmetric Canonical Geometry

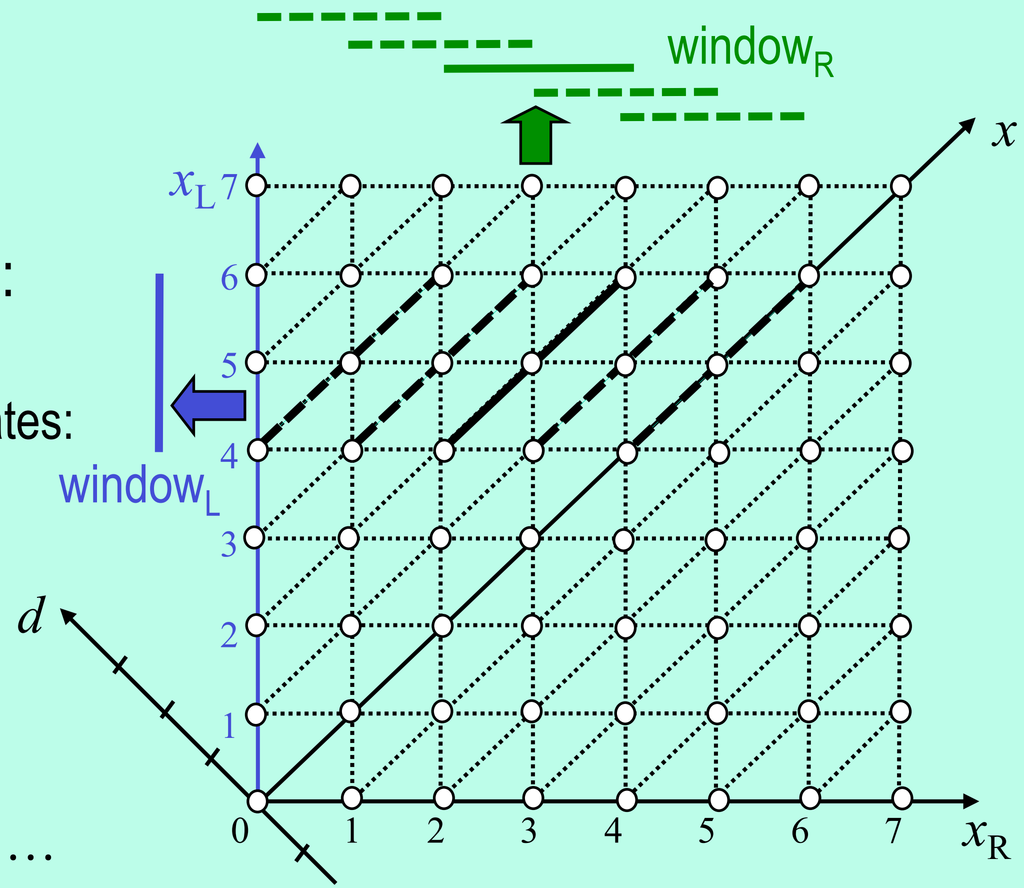
- Symmetric coordinates: $[x,y] \leftrightarrow [x_L,y], [x_R,y]$
  - Disparity: $d = x_L - x_R$
- Cyclopean disparity map: (x,y,d)
  - Symmetric (x,d)-coordinates:

$$\begin{pmatrix} x = (x_L + x_R)/2 \\ d = x_L - x_R \end{pmatrix} \leftrightarrow \begin{pmatrix} x_L = x + d/2 \\ x_R = x - d/2 \end{pmatrix}$$

$$x_L, x_R = 0,\ 1,\ 2,\ \ldots$$

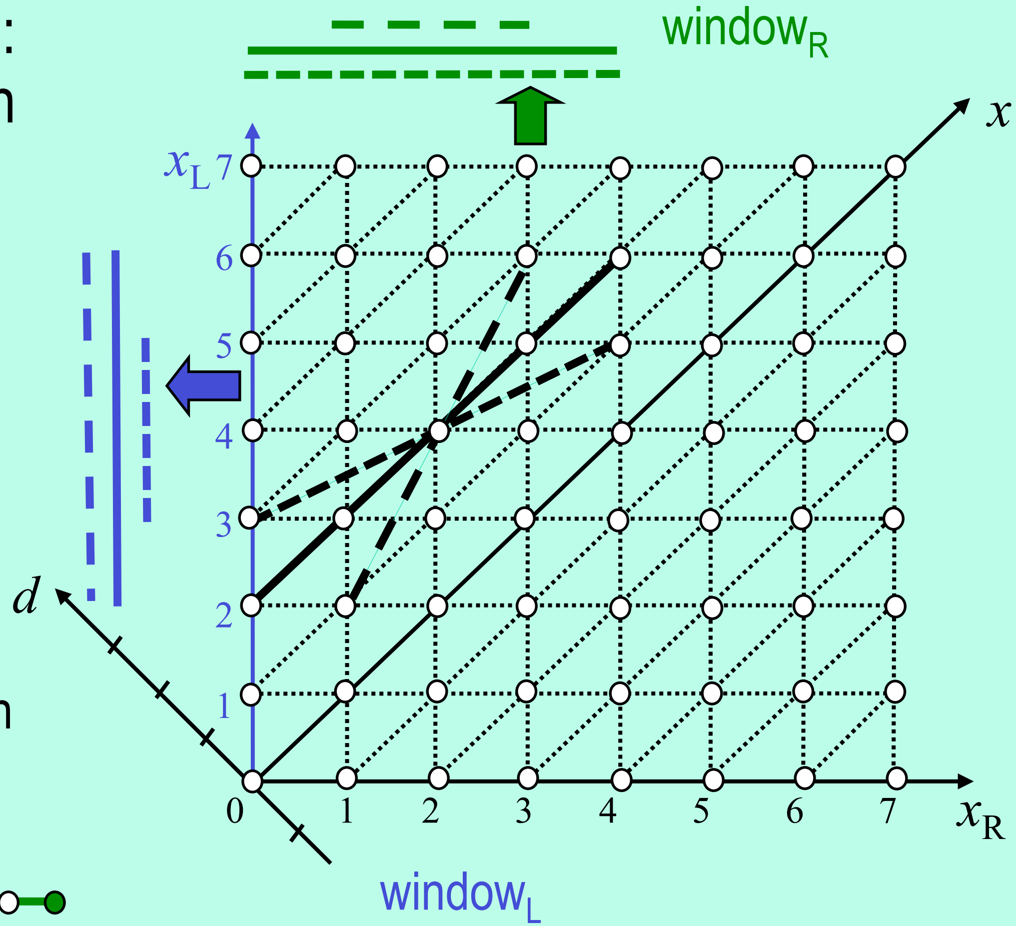
$$x = 0,\ \tfrac{1}{2},\ 1,\ 1\tfrac{1}{2},\ 2,\ \ldots$$

$$d = \ldots,\ -2,\ -1,\ 0,\ 1,\ 2,\ \ldots$$



Matching Slant Planar Surfaces

- **Frontal planar surfaces**: the same window sizes in both images
- **Slant planar surfaces**: the windows of different size in both images
  - Rotating a planar surface patch to specify the size of the window in each image
  - **Interpolation** or repetition of image signals to match points of the windows



Simple Signal Models: 1

- No local geometric distortion (in windows of fixed size)
  - i.e. patches of a frontal planar 3-D surface
- No photometric distortion except from a random noise:
  - Independent centred pixel-wise noise n with a fixed variance

$$g_{\mathrm{L}:x+\frac{d}{2},y} = g_{x,y} + n_{\mathrm{L}:x+\frac{d}{2},y};\quad g_{\mathrm{R}:x-\frac{d}{2},y} = g_{x,y} + n_{\mathrm{R}:x-\frac{d}{2},y}$$

$$\Rightarrow g_{\mathrm{L}:x+\frac{d}{2},y} = g_{\mathrm{R}:x-\frac{d}{2},y} + n_{x,y,d}$$

$$\Rightarrow D_{x,y,d} = \begin{cases} \sum_{i=-a}^{a}\sum_{j=-b}^{b} \left| g_{\mathrm{L}:x+\frac{d}{2}+i,y+j} - g_{\mathrm{R}:x-\frac{d}{2}+i,y+j} \right| \\ \sum_{i=-a}^{a}\sum_{j=-b}^{b} \left(g_{\mathrm{L}:x+\frac{d}{2}+i,y+j} - g_{\mathrm{R}:x-\frac{d}{2}+i,y+j} \right)^2 \end{cases}$$

Unknown signal for the 3-D point

Independent random pixel-wise noise with the same distribution

Independent random pixel-wise noise with the same distribution: $n_{x,y,d} = n_{\mathrm{L}:x+\frac{d}{2},y} - n_{\mathrm{R}:x-\frac{d}{2},y}$

SSD / SAD Based Matching

- **Sum of squared distances** (SSD):

  Symmetric cyclopean case: $D_{x,y,d} = \sum_{i=-a}^{a} \sum_{j=-b}^{b} \left(g_{\mathrm{L}:x+\frac{d}{2}+i,y+j} - g_{\mathrm{R}:x-\frac{d}{2}+i,y+j} \right)^2$

  Asymmetric conventional case: $D_{x',y';x,y} = \sum_{i=-a}^{a} \sum_{j=-b}^{b} \left(g_{\mathrm{L}:x'+i,y'+j} - g_{\mathrm{R}:x+i,y+j} \right)^2$

- **Sum of absolute distances** (SAD):

  Symmetric cyclopean case: $D_{x,y,d} = \sum_{i=-a}^{a} \sum_{j=-b}^{b} \left| g_{\mathrm{L}:x+\frac{d}{2}+i,y+j} - g_{\mathrm{R}:x-\frac{d}{2}+i,y+j} \right|$

  Asymmetric conventional case: $D_{x',y';x,y} = \sum_{i=-a}^{a} \sum_{j=-b}^{b} \left| g_{\mathrm{L}:x'+i,y'+j} - g_{\mathrm{R}:x+i,y+j} \right|$

Simple Signal Models: 2

- No local geometric distortions (in windows of fixed size)
  - i.e. a patch of a frontal planal 3-D surface
- Uniform contrast and offset photometric distortions
  - Independent centred noise n with a fixed variance
  - Fixed contrast factor α and offset β for every window
- Asymmetric left-to-right signal model:

$$g_{\mathrm{L}:x',y'} = \alpha g_{\mathrm{R}:x,y} + \beta + n_{x,y} \Rightarrow D_{x',y';x,y} = \min_{\alpha,\beta} D_{x',y';x,y;\alpha,\beta}$$

$$= \min_{\alpha,\beta} \sum_{i=-a}^{a} \sum_{j=-b}^{b} \left(g_{\mathrm{L}:x'+i,y'+j} - \alpha g_{\mathrm{R}:x+i,y+j} - \beta \right)^2$$

Deriving the Correlation Score

$$\min_{\alpha,\beta} D_{x',y';x,y;\alpha,\beta} = \min_{\alpha,\beta} \sum_{i=-a}^{a} \sum_{j=-b}^{b} \left(g_{\mathrm{L}:x'+i,y'+j} - \alpha g_{\mathrm{R}:x+i,y+j} - \beta \right)^2 \Rightarrow \begin{cases} \left. \dfrac{\partial D_{x',y';x,y;\alpha,\beta}}{\partial \beta} \right|_{\alpha=\alpha^*,\beta=\beta^*} = 0 \\ \left. \dfrac{\partial D_{x',y';x,y;\alpha,\beta}}{\partial a} \right|_{\alpha=\alpha^*,\beta=\beta^*} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \sum_{i=-a}^{a} \sum_{j=-b}^{b} \left(g_{\mathrm{L}:x'+i,y'+j} - \alpha^* g_{\mathrm{R}:x+i,y+j} - \beta^* \right) = 0 \\ \sum_{i=-a}^{a} \sum_{j=-b}^{b} \left(g_{\mathrm{L}:x'+i,y'+j} - \alpha^* g_{\mathrm{R}:x+i,y+j} - \beta^* \right) g_{\mathrm{R}:x+i,y+j} = 0 \end{cases} \Rightarrow \begin{cases} \beta^* = \dfrac{\sum_{i=-a}^{a} \sum_{j=-b}^{b} g_{\mathrm{L}:x'+i,y'+j} - \alpha^* \sum_{i=-a}^{a} \sum_{j=-b}^{b} g_{\mathrm{R}:x+i,y+j}}{(2a+1)(2b+1)} \\ \sum_{i=-a}^{a} \sum_{j=-b}^{b} \left(g_{\mathrm{L}:x'+i,y'+j} - \alpha^* g_{\mathrm{R}:x+i,y+j} - \beta^* \right) g_{\mathrm{R}:x+i,y+j} = 0 \end{cases}$$

Deriving the Correlation Score

Therefore, $\beta^* = m_{\mathrm{L}:x',y'} - \alpha^* m_{\mathrm{R}:x,y};\ \ \alpha^* = \frac{S_{\mathrm{LR}:x',y';x,y}}{S_{\mathrm{RR}:x',y';x,y}}$ where

$$m_{\mathrm{L}:x',y'} = \frac{S_{\mathrm{L}:x',y'}}{S_0};\ m_{\mathrm{R}:x,y} = \frac{S_{\mathrm{R}:x,y}}{S_0};$$

Mean signals in the windows

Window size

$$S_{\mathrm{L}:x',y'} = \sum_{i=-a}^{a}\sum_{j=-b}^{b} g_{\mathrm{L}:x'+i,y'+j};\ \ S_{\mathrm{R}:x,y} = \sum_{i=-a}^{a}\sum_{j=-b}^{b} g_{\mathrm{R}:x+i,y+j};\ \ S_0 = (2a+1)(2b+1)$$

Cross-product of the centred signals in the windows

$$S_{\mathrm{LR}:x',y';x,y} = \sum_{i=-a}^{a}\sum_{j=-b}^{b}\left(g_{\mathrm{L}:x'+i,y'+j} - m_{\mathrm{L}:x',y'}\right)\left(g_{\mathrm{R}:x+i,y+j} - m_{\mathrm{R}:x,y}\right)$$

$$S_{\mathrm{RR}:x,y} = \sum_{i=-a}^{a}\sum_{j=-b}^{b}\left(g_{\mathrm{R}:x+i,y+j} - m_{\mathrm{R}:x,y}\right)^2$$

Non-normalised signal variance in the window on $\mathbf{g}_{\mathrm{R}}$

Deriving the Correlation Score

- Minimum distance between the signals in the windows under the optimum relative contrast α^* and offset β^*:

$$D_{x',y';x,y} = S_{\mathrm{LL}:x',y'} - \frac{S^2_{\mathrm{LR}:x',y';x,y}}{S_{\mathrm{RR}:x,y}} = S_{\mathrm{LL}:x',y'}\left(1 - \frac{S^2_{\mathrm{LR}:x',y';x,y}}{S_{\mathrm{LL}:x',y'}S_{\mathrm{RR}:x,y}}\right)$$

where $S_{\mathrm{LL}:x',y'} = \sum_{i=-a}^{a}\sum_{j=-b}^{b}\left(g_{\mathrm{L}:x'+i,y'+j} - m_{\mathrm{L}:x',y'}\right)^2$

Non-normalised signal variance in the window on $\mathbf{g}_\mathrm{L}$

$$D_{x',y';x,y} = S_{\mathrm{LL}:x',y'}\left(1 - C^2_{x',y';x,y}\right) \in [0,\ S_{\mathrm{LL}:x',y'}]$$

$$C_{x',y';x,y} = S_{\mathrm{LR}:x',y';x,y} \Big/ \sqrt{S_{\mathrm{LL}:x',y'}S_{\mathrm{RR}:x,y}} \in [-1,\ 1]$$

Cross-correlation

Correlation Matching

- Cross-correlation $C_{x',y';x,y}$ is most frequently used as the similarity measure (rather than the distance $D_{x',y';x,y}$ as the dissimilarity measure)
- Individual correlation search for a point corresponding to (x',y') in $\mathbf{g}_L$ in a search region SR:

$$(x^*,y^*) = \arg\min_{(x,y)\in \text{SR}} D_{x',y';x,y} \quad \text{or} \quad (x^*,y^*) = \arg\max_{(x,y)\in \text{SR}} C_{x',y';x,y}$$

  - The disparity vector for (x',y') in $\mathbf{g}_L$:

$$\mathbf{d}_{x',y'} = [d_{x',y'} = x' - x^*,\ \delta_{x',y'} = y' - y^*]$$

Constraints on Contrast

- Relative contrast α^* and signal variances $S_{\mathrm{LL}:x',y'}$ and $S_{\mathrm{RR}:x,y}$ should be restricted
  - e.g. $\alpha_{\min} \le \alpha^* \le \alpha_{\max}$ and $S_{\mathrm{LL}:x',y'} \ge \theta > 0$; $S_{\mathrm{RR}:x,y} \ge \theta > 0$ to exclude inadequate matches of almost uniform windows

$$D_{x',y',x,y} = \begin{cases} S_{\mathrm{LL}:x',y'} - \dfrac{S^2_{\mathrm{LR}:x',y',x,y}}{S_{\mathrm{RR}:x,y}} + S_{\mathrm{RR}:x,y}\left(\alpha_{\min} - \dfrac{S_{\mathrm{LR}:x',y',x,y}}{S_{\mathrm{RR}:x,y}}\right)^2 & \text{if } \dfrac{S_{12:x',y',x,y}}{S_{22:x,y}} < \alpha_{\min} \\ S_{\mathrm{LL}:x',y'} - \dfrac{S^2_{\mathrm{LR}:x',y',x,y}}{S_{\mathrm{RR}:x,y}} & \text{if } \alpha_{\min} \le \dfrac{S_{12:x',y',x,y}}{S_{22:x,y}} \le \alpha_{\max} \\ S_{\mathrm{LL}:x',y'} - \dfrac{S^2_{\mathrm{LR}:x',y',x,y}}{S_{\mathrm{RR}:x,y}} + S_{\mathrm{RR}:x,y}\left(\dfrac{S_{\mathrm{LR}:x',y',x,y}}{S_{\mathrm{RR}:x,y}} - \alpha_{\max}\right)^2 & \text{if } \alpha_{\max} < \dfrac{S_{12:x',y',x,y}}{S_{22:x,y}} \end{cases}$$

Fast Implementation

- Window-size independent sums $Q_{x,y} = \sum_{i=-a}^{a} \sum_{j=-b}^{b} q_{x+i,y+j}$
  - Accumulator: $\mathbf{Q}^{\text{acc}} = \left\{ Q^{\text{acc}}_{\xi,\eta} = \sum_{x=0}^{\xi} \sum_{y=0}^{\eta} q_{x,y} \,\middle|\, x = 0,\ldots,M-1; y = 0,\ldots,N-1 \right\}$

    $$\Leftrightarrow Q^{\text{acc}}_{\xi,\eta} = q_{\xi,\eta} + Q^{\text{acc}}_{\xi-1,\eta} + Q^{\text{acc}}_{\xi,\eta-1} - Q^{\text{acc}}_{\xi-1,\eta-1}$$

  - Window sum: $Q_{x,y} = Q^{\text{acc}}_{x+a,y+b} - Q^{\text{acc}}_{x-a-1,y+b} - Q^{\text{acc}}_{x+a,y-b-1} + Q^{\text{acc}}_{x-a-1,y-b-1}$
- Straightforward correlation matching: $O(MNab\Delta)$
- Fast correlation matching: $O(MN\Delta)$
  - Δ - disparity range; ab - the dominant term of the window size
    - Even for a small window 11 x 11 pixels: ~ 120 times faster!

Window Sums

Using the accumulated sums: $62 - 40 - 23 + 15$ = **14**

$0+0+1+2+0+0+1+1+0+0+1+2+0+0+2+1+0+0+1+2$ = **14**

2	2	1	0	0	0	0	1	1	2
2	2	1	0	0	0	0	1	2	1
2	2	1	0	0	0	0	2	1	1
2	2	1	0	0	0	0	1	2	1
2	2	1	0	0	0	0	1	1	2
2	2	1	0	0	0	0	1	2	1
2	2	1	0	0	0	0	2	1	1
2	2	1	0	0	0	0	1	2	1
2	2	1	0	0	0	0	1	1	2
2	2	1	0	0	0	0	1	2	1

2	4	5	5	5	5	5	6	7	9
4	8	10	10	10	10	10	12	15	18
6	12	15	15	15	15	15	19	23	27
8	16	20	20	20	20	20	25	31	36
10	20	25	25	25	25	25	31	38	45
12	24	30	30	30	30	30	37	46	54
14	28	35	35	35	35	35	44	54	63
16	32	40	40	40	40	40	50	62	72
18	36	45	45	45	45	45	56	69	80
20	40	50	50	50	50	50	62	77	89

Fast Implementation

- $S_{\mathrm{L}:x',y'}$ and $S_{\mathrm{R}:x,y}$: $q_{x,y} = g_{\mathrm{L}:x,y}$ or $g_{\mathrm{R}:x,y}$
- $S_{\mathrm{LL}:x',y'}$ and $S_{\mathrm{RR}:x,y}$: $q_{x,y} = g^2_{\mathrm{L}:x,y}$ or $g^2_{\mathrm{R}:x,y}$

$$S_{\mathrm{LL}:x',y'} = \sum_{i=-a}^{a} \sum_{j=-b}^{b} \left(g_{\mathrm{L}:x'+i,y'+j} - m_{\mathrm{L}:x',y'} \right)^2 = \sum_{i=-a}^{a} \sum_{j=-b}^{b} g^2_{\mathrm{L}:x'+i,y'+j} - S_0 m^2_{\mathrm{L}:x',y'}$$

- $S_{\mathrm{LR}:x',y';\,x,y}$: using an accumulator for each current pair of disparities $d = x' - x;\ \delta = y' - y$
  - Compute the correlation (or distance) for every window under fixed disparities $(d,\ \delta)$ and keep in each location (x',y') both its current optimum and the disparity pair for it

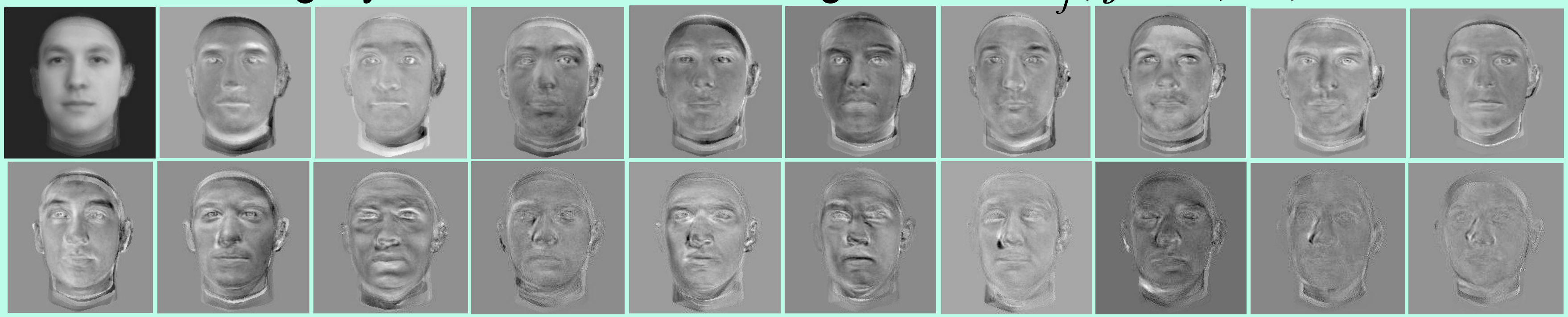
Soft Least-squares Masking

- Least-squares image matching frequently fails in the presence of sizeable areas with large differences between corresponding signals
  - The differing signals are frequently called **outliers**
- We attempt to eliminate outliers by **soft masking** of suspicious pixels in an input image
  - Image differences are modelled with a mixture of a "valid" random noise and outliers
  - **Soft masks** with weights in [0.0,1.0] for each pixel: by an Expectation-Maximisation (EM) procedure

MIT Face Database Subset

Co-registered images //vismod.media.mit.edu/pub/images/

Centroid and grey-coded normalised eigen-faces $\mathbf{e}_j; j = 1,\ldots,19$



MIT Face Database Subset

- Ten subjects (20 co-registered 200 x 200 images)
- Two images of every subject under different illumination
  - Eigen-values λ_j and cumulative relative variances of signals $\nu_j = \sum_{i=1}^{j} \lambda_i \Big/ \sum_{i=1}^{19} \lambda_i$ represented by the j top eigen-faces:

	$\mathbf{e}_1$	$\mathbf{e}_2$	$\mathbf{e}_3$	$\mathbf{e}_4$	$\mathbf{e}_5$	$\mathbf{e}_6$	$\mathbf{e}_7$	$\mathbf{e}_8$	$\mathbf{e}_9$
$\lambda_j \times 10^{-7}$	4.5	1.4	0.79	0.64	0.40	0.33	0.27	0.22	0.15
$\nu_j(\%)$	50	67	76	83	87	91	94	96	98

Error probability mixture model

$$\Pr(\varepsilon_i) = \rho N(\varepsilon_i|\sigma) + (1-\rho)U(\varepsilon_i);\ \Pr(\boldsymbol{\varepsilon}) = \Pr(\varepsilon_1)\cdot\ldots\cdot\Pr(\varepsilon_p)$$

implies that noise can be separated from outliers by masking out pixels with significantly larger than expected errors

Additional notation:

Q - the number of grey levels (typically, $Q = 256$)

Discrete errors $\varepsilon \in E = \{-Q+1, \ldots, -1, 0, 1, \ldots, Q-1\}$

ρ - unknown prior probability of non-outliers

$U(\varepsilon)$ - uniform, by assumption, distribution of outlying errors: $U(\varepsilon) = \frac{1}{2Q-1}$

$N(\varepsilon|\sigma)$ - discrete truncated zero-centred Gaussian distribution:

$$N(\varepsilon|\sigma) = \frac{1}{Z_\sigma}\exp\left(-\frac{\varepsilon^2}{2\sigma^2}\right) \quad \text{where} \quad Z_\sigma = \sum_{\delta=-Q+1}^{Q-1}\exp\left(-\frac{\delta^2}{2\sigma^2}\right)$$

Robust Symmetric Matching

- $\tilde{\mathbf{g}}_1, \tilde{\mathbf{g}}_2$: images derived from an unknown template $\tilde{\mathbf{g}}$ by varying uniform contrast (a_1 and a_2) and offset (b_1 and b_2) values; $\theta = (a_1, a_2, b_1, b_2, \tilde{\mathbf{g}})$
- Images are perturbed by independent per pixel errors ε caused by **noise** or **outlier**: for noise only, $\tilde{g}_{1i} = a_1\tilde{g}_i + b_1 + \varepsilon_{1i}; \quad \tilde{g}_{2i} = a_2\tilde{g}_i + b_2 + \varepsilon_{2i}$

- **Maximum likelihood signal dissimilarity** of images in the presence of noise and outliers:

$$D_{12} = \min_{\theta}\left(-\ln \Pr(\tilde{\mathbf{g}}_1, \tilde{\mathbf{g}}_2)\right) = -\sum_{i=1}^{p}\left[\gamma_i \ln N\left(\varepsilon_i | \sigma\right) + (1-\gamma_i)\ln U\left(\varepsilon_i\right)\right]$$

$$= \frac{1}{2\sigma^2}\Phi_{12} + \nu \ln\left(Z_\sigma\right) + (p-\nu)\ln(2Q-1) \quad \text{where} \quad \nu = \sum_{i=1}^{p}\gamma_i$$

Robust Symmetric Matching

- Φ_{12} - minimum total squared error with respect to model parameters θ:

$$\Phi_{12} = \min_{\theta} \sum_{i=1}^{p} \gamma_i \left(\varepsilon_{1i}^2 + \varepsilon_{2i}^2 \right) = \tfrac{1}{2}\left[S_{11} + S_{22} - \sqrt{(S_{11} - S_{22})^2 + 4S_{12}^2} \right]$$

where

$$S_{kl} = \sum_{i=1}^{p} \gamma_i \left(\tilde{g}_{ki} - \mu_k \right)\left(\tilde{g}_{li} - \mu_l \right); \quad k,l = 1,2; \quad \mu_k = \tfrac{1}{\nu} \sum_{i=1}^{p} \gamma_i \tilde{g}_{ki}$$

- $\varepsilon_i = \alpha_2(g_{1i} - \mu_1) - \alpha_1(g_{2i} - \mu_2)$ - residual per pixel matching error after the ML estimation and elimination of model parameters:

$$\alpha_1^2 = \frac{1}{2}\left(1 + \frac{S_{11} - S_{22}}{\sqrt{(S_{11} - S_{22})^2 + 4S_{12}^2}} \right); \quad \alpha_2^2 = \frac{1}{2}\left(1 - \frac{S_{11} - S_{22}}{\sqrt{(S_{11} - S_{22})^2 + 4S_{12}^2}} \right)$$

Robust Symmetric Matching

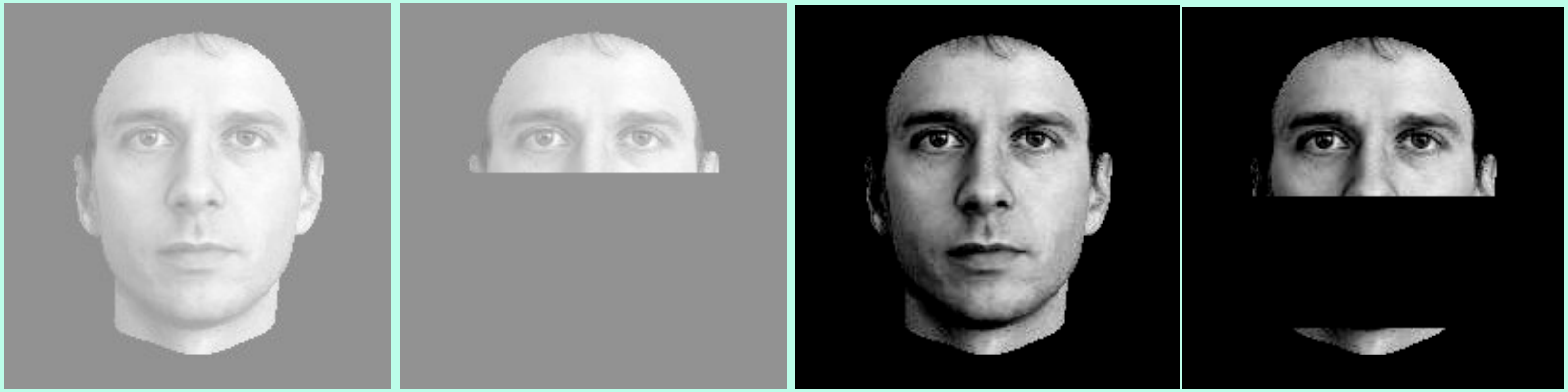
- Estimated noise variance:

$$\sigma^2 = \frac{\Phi_{12}}{2\nu} \quad \text{so that} \quad D_{12} = \nu \ln\left(1 + \sum_{\delta=1}^{Q-1} 2\exp\left(-\frac{\delta^2 \nu}{\Phi_{12}}\right)\right) + \nu + (p - \nu)\ln(2Q - 1)$$

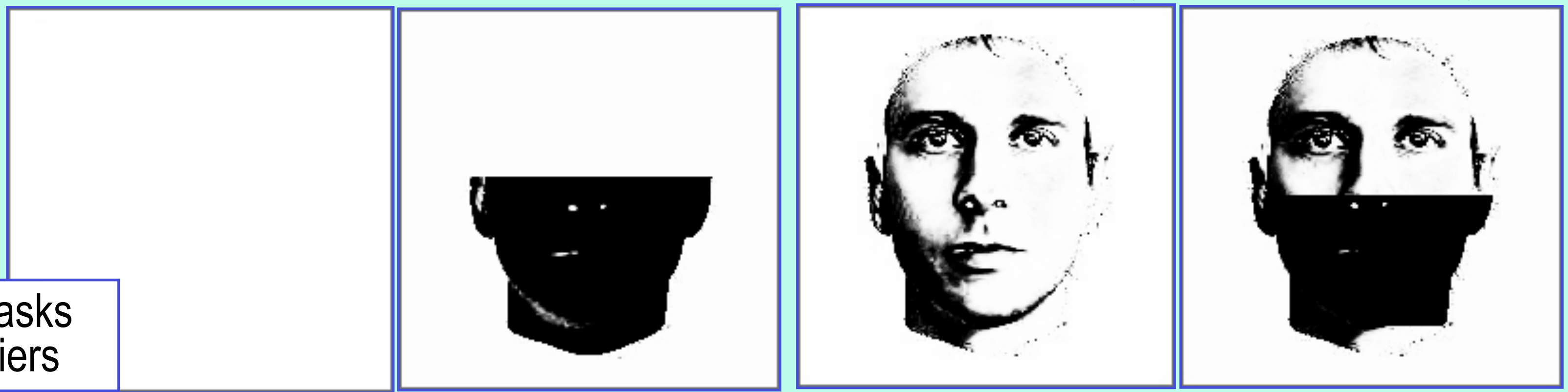
- Local minimum of Φ_{12} :
  - by EM-based iterative procedure
    - Re-evaluates soft masks γ and model parameters θ, similar to the robust PCA-based reconstruction algorithm

Robust Symmetric Matching

Face #1: Linear contrast + occlusion | Non-linear contrast + occlusion



First match: $D_{12}\times10^{-5}$	01: 0.40	01: 1.11	01: 1.41_4	01: 1.47_9
Second match:	02: 1.08	02: 1.32	02: 1.41_5	02: 1.49_5



Soft masks of outliers

Robust Symmetric Matching

Input: Images $\tilde{\mathbf{g}}_1$ and $\tilde{\mathbf{g}}_2$ to be matched.

Initial step $t = 0$: Match images with unit mask $\boldsymbol{\gamma}^{[0]} = [1,\ldots,1]$ to find:

the conventional symmetric matching score $D_{12}^{[0]}$ and

initial values $\mathbf{\Phi}_{12}^{[0]},\ \sigma_{[0]}^2 = \frac{1}{2p}\mathbf{\Phi}_{12}^{[0]},\ \alpha_1^{[0]},\ \alpha_2^{[0]},\ \mu_1^{[0]},\ \mu_2^{[0]};$

Set prior $\rho^{[0]} = 0.5$

Iteration $t = 1, 2, \ldots$: Reset mask and prior for current residual errors

$\varepsilon_i^{[t]} = \alpha_2^{[t-1]}\left(\tilde{g}_{1i} - \mu_1^{[t-1]}\right) - \alpha_1^{[t-1]}\left(\tilde{g}_{2i} - \mu_2^{[t-1]}\right)$:

$$\gamma_i^{[t]} = \frac{\rho_{[t-1]} N\left(\varepsilon_i^{[t]} \middle| \sigma_{[t-1]}\right)}{\rho_{[t-1]} N\left(\varepsilon_i^{[t]} \middle| \sigma_{[t-1]}\right) + \left(1 - \rho_{[t-1]}\right) U\left(\varepsilon_i^{[t]}\right)}; \quad \nu_{[t]} = \sum_{i=1}^{p} \gamma_i^{[t]}; \quad \rho_{[t]} = \frac{\nu_{[t]}}{p}$$

and update $S_{kl}^{[t]};\ \mu_k^{[t]};\ k,l = 1,2;\quad \mathbf{\Phi}_{12}^{[t]},\ \sigma_{[t]}^2 = \frac{1}{2\nu_{[t]}}\mathbf{\Phi}_{12}^{[t]},\ \ \alpha_1^{[0]},\ \ \alpha_2^{[0]}$

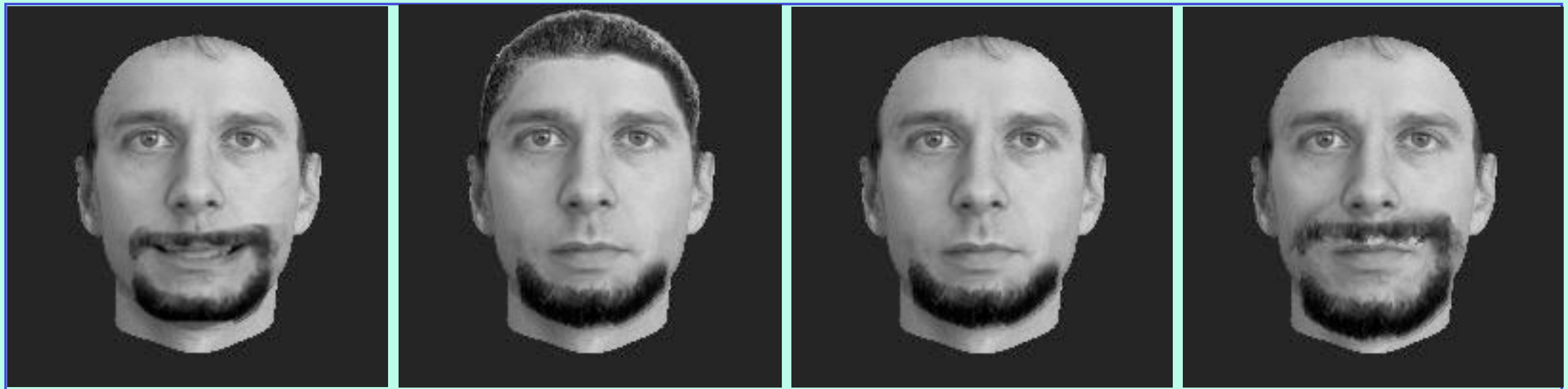
Stopping rule: Terminate if $\left| D_{12}^{[t]} - D_{12}^{[t-1]} \right| \leq \theta_r D_{12}^{[t]}$ or $t > \theta_i$

where θ_r and θ_i are fixed thresholds

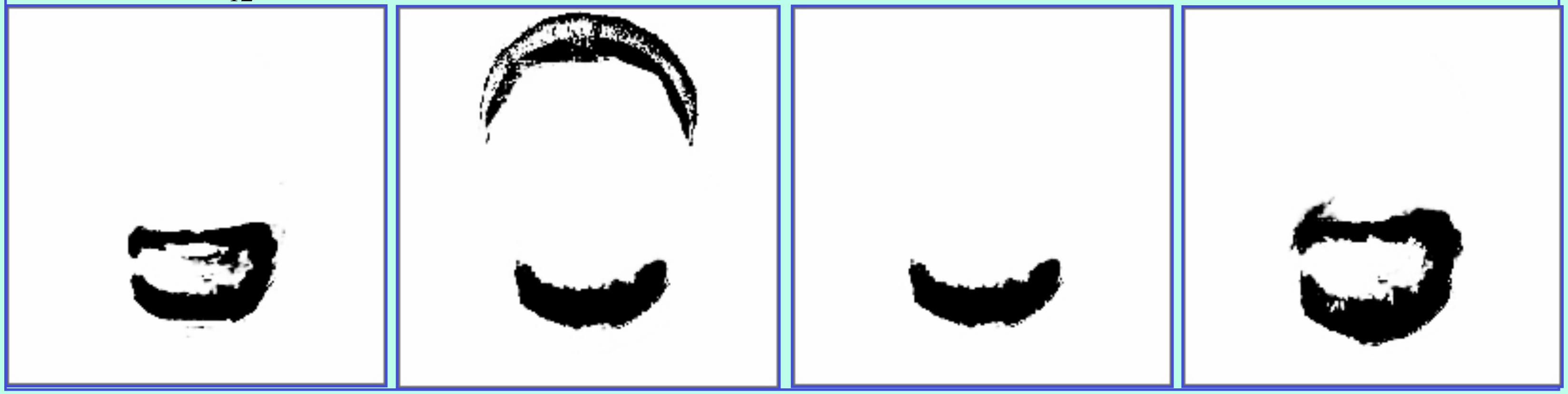
Distorted Face #1: Soft Masks

First match: D_{12} 01: 1.03 | 01: 1.35 | 01: 1.39 | 01: 1.30

Distorted Face #1: Soft Masks



First match: D_{12} 01: 0.96 | 01: 1.08 | 01: 0.80 | 01: 1.02

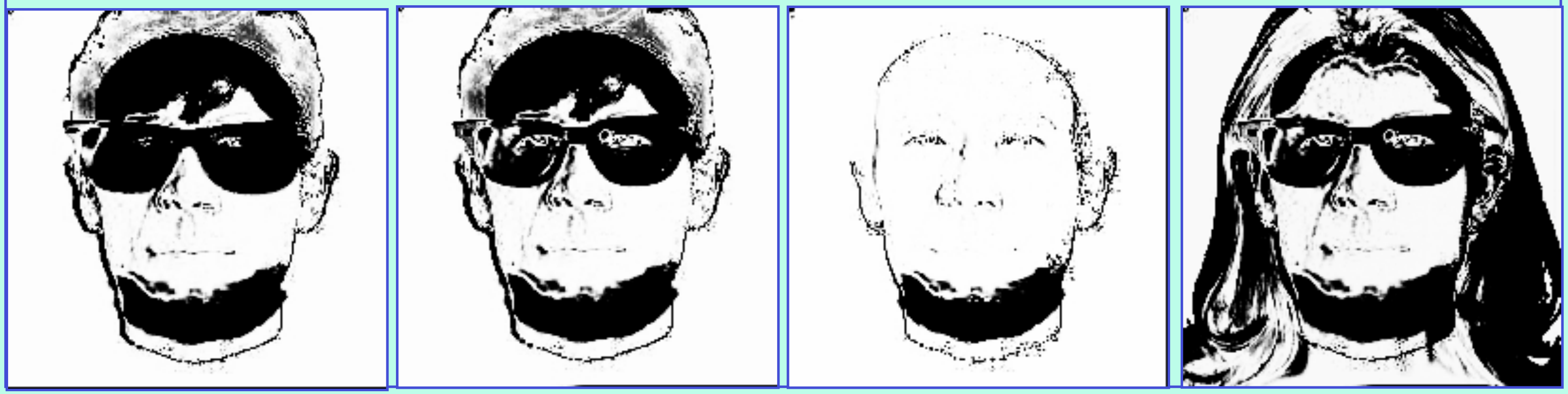


Distorted Face #1: Soft Masks

First match: D_{12} 01: 1.51 | 01: 1.50 | 01: 1.85 | 01: 1.73

Distorted Face #3: Soft Masks

First match: D_{12} 03: 1.61 | 03: 1.60 | 03: 1.27 | 03: 1.90



Distorted Faces #3,9: Soft Masks

First match: D_{12} 03: 1.49 03: 1.50 03: 1.83 09: 1.51

Distorted Face #9: Soft Masks

First match: D_{12} 09: 1.72 09: 1.80 09: 1.79 09: 1.99

Soft Masking: Matching Results

- Soft masking of outliers significantly increases the accuracy of image matching

Error rates for the symmetric LS matching:	*Conventional*	*Robust*
24 distorted MIT FDB faces (Slides 19 - 24)	54%	0%
40 half-occluded MIT FDB faces (*black top or bottom half*)	47%	5%

  – If models account for contrast and offset deviations and outliers, images can be matched or reconstructed more effectively even when large image regions are modified

- ***Directions of subsequent research***:
  – More adequate probability models of noise and outliers
  – Embedding the EM-based matching algorithms into a general least squares framework to account for geometric deviations

Least-Squares Correlation

- Search for geometric transformations $\mathbf{t}^*$ maximising the cross-correlation matching score between the windows:

$$C_{\mathbf{t}^*} = \max_{\mathbf{t}} \{ C_{\mathbf{t}} \}$$

- Simplified case: affine transformations $\begin{cases} x_R = a_1 x_L + a_2 y_L + a_3 \\ y_R = a_4 x_L + a_5 y_L + a_6 \end{cases}$
  - Combined exhaustive and directed (e.g. gradient based) search for affine parameters
  - Exhaustion of a sparse grid of the relative translations a_3 and a_5 of the fixed window g_1 with respect to the other image g_2
  - Directed unconstrained optimisation of $C_{\mathbf{t}}$ by all $\mathbf{t} = [a_1,\ldots,a_6]$ affine parameters starting from each grid point $\mathbf{t} = [1,0,a_3,0,1,a_6]$

Affinely Transformed Windows

M15

Transformed M28

M28

Transformed M15

M24

Transformed M25

M25

Transformed M24

M29

Transformed M30

M30

Transformed M29