Learning Sets of Rules

Computer Science 760
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Motivation

- Set of if-then rules that jointly define the target function
- Rules are easy (?) for people to understand and edit
- Rules we've seen
  - Translate a decision tree into a set of rules
  - Use a genetic algorithm that encodes a rule set
- But also first-order rules or partial or overlapping models

Sequential Covering

- Learn one-rule, remove the data it covers, then iterate
- Our rule must have high accuracy but not necessarily high-coverage (what does this do to the overfitting/oversearching problem??)
- Only throw out positive examples covered
- Final rules sorted by accuracy over the *whole* training set
- Widely used

Issues with Sequential Covering

- Greedy search so no guarantees about smallest set or best set of rules
- So each rule is learned on a different distribution of the training set…..isn't this a problem???
- Definitely skewed to best "set of rules" not best "rules"

Sequential Covering Algorithm

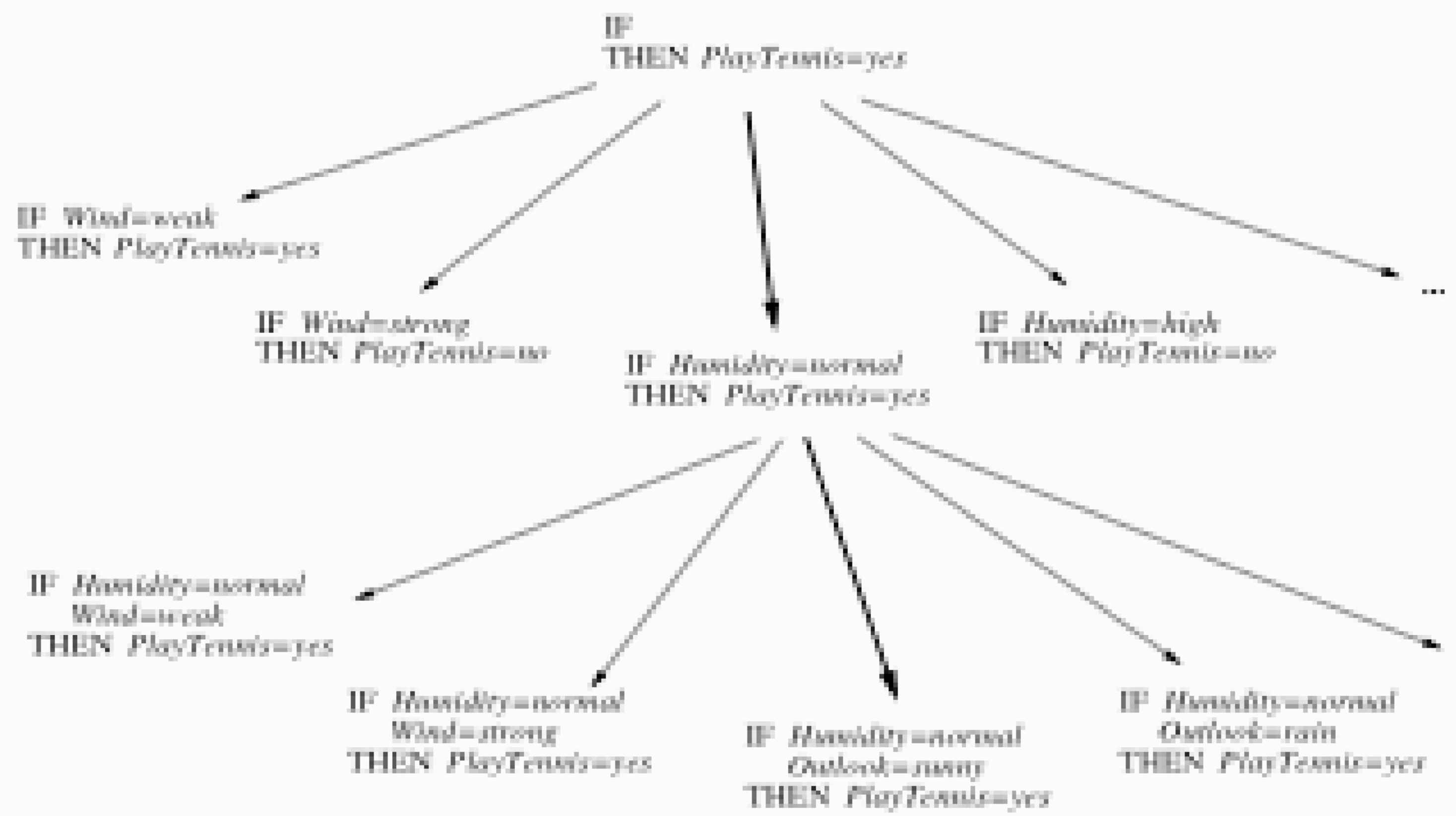
Sequential-covering(Target-attribute, Attributes, Examples, Threshold)

- Learned_rules←{}
- Rule←LEARN-ONE-RULE(Target-attribute, Attributes, Examples)
- While PERFORMANCE(Rule, Examples) > Threshold, do
  - Learned-rules←Learned-rules + Rule
  - Examples←Examples-{examples correctly classified by Rule}
  - Rule←LEARN-ONE-RULE(Target-attribute, Attributes, Examples)
- Learned-rules ←sort Learned-rules according to PERFORMANCE over Examples
- Return Learned-rules

How to Learn-One-Rule

- General-to-specific search through the space of possible rules in search of a rule with high accuracy
- Many ways to evaluate best descendant (same as decision trees) - like entropy
- (greedy, no-backtracking) can extend to beam-search - CN2
- Search continues until it reaches a maximally specific hypothesis that contains all available attributes
- Postcondition is determined last

General to Specific Beam Search



Variation

- Learning rules for only a single class - negation as failure - “pregnant women who are likely to have twins”
- Must change “performance” to fractions of positives covered - AQ - Skewed sample size encourages this also!
- AQ uses single positive seed example to focus search in Learn-One-Rule
- Only considers attributes satisfied by that positive instance
- A new seed example is chosen from those positive examples not yet covered

Design Choices: Sequential versus Simultaneous

- Sequential Covering Algorithms learn one rule at a time, remove the covered examples, and repeat.
- Decision trees can be seen as Simultaneous Covering Algorithms
- Sequential covering algorithms perform n*k primitive search steps to learn n rules each containing k attribute-value tests. If the decision trees is a complete binary tree, it makes (n-1) primitive search steps where n is the number of paths (i.e., rules).
- So Sequential Covering Algorithms must be supported by additional data, but have the advantage of allowing rules with different tests.

General-to-Specific versus Specific-to-General

- General to specific starts at the one maximally general hypothesis
- In specific to general there are many maximally specific hypothesis (the training data).
- Golem chooses several randomly and picks the best learned hypothesis.

Generate-then-test or Example-driven

- GTT hypothesis performance is based on many training examples
- the effect of noisy data is minimized

Post-pruning

- In either system post-pruning can be used to increase the effectiveness of rules on a validation set

Rule Performance Measures

- Relative frequency - AQ - $\frac{n_c}{n}$
- M-estimate of accuracy - CN2 - $\frac{N_c + mp}{n + m}$
- Entropy - CN2 - $-Entropy(S) = \sum_{i=1}^{c} p_i \log_2 p_i$

Exhaustive Rule Learning

- Greedy search can miss good rules
  - What about over-searching???
    - Really multiple comparison problem
- Disallowing overlapping rules can cause problems
- Solution: look at every rule and keep it if it is good

Brute

- Exhaustive depth bounded search
- When evaluating single rules coverage is important
  - Chi-squared statistic
- Multiple comparisons more of a problem!!
  - Validation sets difficult for rules
  - We use randomization testing
- Presenting multiple rules are difficult
  - Also a problem with similar rules and additional conjuncts
- “equivalent to” association rules

Brute Run

```
> brute -T iopus -d 4 -S chi -F simnum -F simparent -r 100 dataset3 b
Setting up tests...
Doing search...
   1: MinPos = 1, Tests = 1899 .... Rules = 1,899, Seconds = 1.
   2: MinPos = 1, Tests = 1899 .... Rules = 168,025, Seconds = 1.
   3: MinPos = 1, Tests = 1899 .... Rules = 7,673,351, Seconds = 21.
   4: MinPos = 1, Tests = 1899 .... Rules = 161,432,100, Seconds = 464.
done.

Data positive coverage =  69.2%.
Test positive coverage =  25.0%.

Search time = 486 seconds.
Rules examined = 169,275,375.
Search speed = 348,303 rules per second.
>
```

Brute Top Rules

	Data			Test		
	Acc	Cov	Chi	Acc	Cov	Chi
IF attr6 = a && attr11 <> e && attr31 >= 21 THEN attr1 = b	100.0	34.6	116.7	50.0	12.5	8.3
IF attr6 <> c && attr6 <> b && attr11 <> e && attr31 >= 24 THEN attr1 = b	100.0	34.6	116.7	50.0	12.5	8.3
IF attr6 <> c && attr7 >= 27 && attr11 <> e && attr31 >= 24 THEN attr1 = b	100.0	34.6	116.7	50.0	12.5	8.3
IF attr2 = c && attr3 <> c && attr7 >= 27 && attr35 < 1029 THEN attr1 = b	78.6	42.3	107.4	0.0	0.0	0.1
IF attr3 <> c && attr7 >= 24 && attr31 >= 21 && attr39 >= 7 THEN attr1 = b	83.3	38.5	104.7	20.0	12.5	2.3
IF attr6 <> c && attr7 >= 24 && attr31 >= 27 && attr39 >= 7 THEN attr1 = b	100.0	30.8	103.7	20.0	12.5	2.3

Brute Bottom Rules

	Data			Test		
	Acc	Cov	Chi	Acc	Cov	Chi
IF attr6 <> c && attr6 <> b && attr25 >= 9 && attr31 < 139 THEN attr1 = b	100.0	23.1	77.8	33.3	12.5	4.9
IF attr6 <> c && attr6 <> b && attr25 >= 9 && attr31 >= 16 THEN attr1 = b	100.0	23.1	77.8	50.0	12.5	8.3
IF attr6 <> c && attr6 <> b && attr25 >= 9 && attr35 < 1029 THEN attr1 = b	100.0	23.1	77.8	0.0	0.0	0.1
IF attr6 <> c && attr6 <> b && attr25 >= 9 && attr35 >= 64 THEN attr1 = b	100.0	23.1	77.8	33.3	12.5	4.9
IF attr6 <> c && attr6 <> b && attr25 >= 9 && attr36 = i THEN attr1 = b	100.0	23.1	77.8	33.3	12.5	4.9
IF attr6 <> c && attr6 <> b && attr25 >= 9 && attr30 = u THEN attr1 = b	100.0	23.1	77.8	33.3	12.5	4.9

Other Brute Features

- Only does one class at a time
- Chi-square allows negative rules to be found
- Can use beam-search
- Can make decision list

Confusion Matrix

	Then True	Then False	
IF True	a	b	N_{IT}
IF False	c	d	N_{IF}
	N_{TT}	N_{TF}	N

Let us look at Accuracy & Error

- $Acc = a/N_{IT}$
- $Error = b/N_{IT}$

Yates Chi Square Formula

$$X^2 = \frac{N(|ad - bc| - \frac{N}{2})^2}{N_{IT} N_{IF} N_{TT} N_{TF}}$$

- Uses the WHOLE table!

Learning First Order Rules

- Inductive logic programming (ILP)
- Automatically inferring Prolog programs from examples

Why Not Propositional Rules?

- Name1=Sharon, Mother1=Louise, Father1=Bob, Male1=False, Female1=True, Name2=Bob, Mother2=Nora, Father2=Victor, Male2=True, Female2=False

- If (Father1=Bob) ^ (Name2=Bob) ^ Female1=True then Daughter1-2=True

- Can't describe relations between attributes!

First Order Horn Clauses

- If Father(y,x) ^ Female(y) then Daughter(x,y)
- Can also have variables in the preconditions which are not used in the postconditions - such variables are assumed to be existentially quantified
  – If Father(y,z) ^ Mother(z,x) ^ Female(y) then GrandDaughter(x,y)
- Can also represent (and learn!) recursive functions
  – If Parents(x,z) ^ Ancestor(z,y) then Ancestor(x,y)

Terminology I

- Every well-formed expression is composed of ***constants*** (e.g., Mary, 23, or Joe), ***variables*** (e.g., x), ***predicates*** (e.g., Female, as in Female(Mary)), and ***functions*** (e.g., age is in age(Mary)).
- A ***term*** is any constant, any variable, or any function applied to any term. Examples include Mary, x, age(Mary), age(x).
- A ***literal*** is any predicate (or its negation) applied to any set of terms. Examples include Female(Mary), ¬Female(x), Greater_than(age(Mary),20)).
- A ***ground literal*** is a literal that does not contain any variables (e.g., ¬Female(Joe)).

Terminology II

- A ***negative literal*** is a literal containing a negated predicate (e.g., ¬Female(Joe)).
- A ***positive literal*** is a literal eith no negation sign (e.g., Female(Mary)).
- A ***clause*** is any disjunction of literals $M_1 v \ldots M_n$ whose variables are universally quantified.
- A ***Horn clause*** is an expression of the form $H \leftarrow (L_1 \wedge \ldots \wedge L_n)$ where H, $L_1 \ldots L_n$ are positive literals. H is called the head or consequent of the Horn clause. The conjunction of literals $L_1 \wedge L_2 \wedge \ldots \wedge L_n$ is called the body or antecedents of the Horn clause.

Terminology III

- For any literals A and B, the expression $(A \leftarrow B)$ is equivalent to $(A \vee \neg B)$, and the expression $\neg(A \wedge B)$ is equivalent to $(\neg A \vee \neg B)$. Therefore a Horn clause can equivalently be written as the disjunction $H \vee \neg L_1 \vee \ldots \vee \neg L_n$.
- A ***substitution*** is any function that replaces variables by terms. For example, the substitution $\{x/3, y/z\}$ replaces the variable x by the term 3 and replaces the variable y by the term z. Given a substitution θ and a literal L we write $L\theta$ to denote the result of applying substitution θ to L.
- A ***unifying substitution*** for two literals L_1 and L_2 is any substitution θ such that $L_1\theta = L_2\theta$.

FOIL

- Extension of Sequential Covering to first order representations
- Learns Horn clauses with 2 exceptions
  1. More restrictive - literals are not permitted to contain function symbols - reduces complexity of hypothesis space
  2. More expressive - literals appearing in the body may be negated
- Learn recursive Quicksort & legal from illegal chess positions

FOIL Algorithm

FOIL(Target-predicate, Predicates, Examples)

- Pos←those Examples for which the Target-predicate is True
- Neg←those Examples for which the Target-predicate is False
- Learned-rules←{}
- While Pos, do

  Learn a NewRule
  - NewRule←the rule that predicts Target-predicate with no preconditions
  - NewRuleNeg←Neg
  - While NewRuleNeg, do

    Add a new literal to specialize NewRule
    - Candidate_literals← generate candidate new literals for NewRule, based on Predicates
    - Best_literal←$\text{argmax}_{L \in \text{Candidate-literals}}$Foil-Gain(L,NewRule)
    - Add Best-literal to preconditions of NewRule
    - NewRuleNeg←subset of NewRuleNeg that satisfies NewRule preconditions
  - Learned-rules←Learned-rules + NewRule
  - Pos←Pos-{members of Pos covered by NewRule}
- Return Learned-rules

Differences between FOIL & Sequential Covering

- Seeks only rules where target literal is True
- Performs simple hill-climbing search rather than beam search
- Adding each new rule generalizes the disjunctive hypothesis so viewed at this level the search is specific-to-general
- Adding new conjuncts to each rule is a general-to-specific hill-climbing search

Issues for FOIL

1. How to generate candidate specializations of a rule - need to accommodate variables

2. What performance measure to use - need to distinguish between different bindings of the rules variables

Generating Candidate Specializations

1. $Q(v_1,\ldots,v_r)$ - where Q is any predicate occurring in Predicates and the v_i are either new variables or variables already present in the rule. At least one v_i in the created literal must already exist in the rule
2. $Equal(x_j,x_k)$ = where x_j and x_k are variables already present in the rule
3. The negation of either of the above

FOIL Example

- GrandDaughter(x,y) where *Predicates* contains Father and Female
- Candidate Literals: Equal(x,y), Female(x), Female(y), Father(x,y), Father(y,x), Father(x,z), Father(z,x), Father(y,z), Father(z,y) and the negation of each
- Let us assume FOIL greedily selects GrandDaugther(x,y) $\leftarrow$ Father(y,z)

FOIL Example II

- FOIL now considers all those before and Female(z), Equal(z,x), Equal(z,y), Father(z,w), Father(w,z) and their negations
- Continues until it covers only positive examples, then remove all positive examples covered and start search for next rule

Guiding Search in FOIL

- Performance of the rule over the training data
- Must consider all possible bindings of each variable
- Use the closed world assumption - any literal involving these predicates and these constants that is not listed is assumed false

Evaluation Function

- Target literal GrandDaughter(x,y)
- Assertions - GrandDaughter(Victor,Sharon), Father(Sharon,Bob), Father(Tom,Bob), Female(Sharon), Father(Bob,Victor)
- Given the 4 constants there are 16 possible variable bindings for the initial rule - 1 positive x/Victor,y/Sharon and 15 negative
- Evaluation function let R´ be the rule created by adding a new literal L to the old rule R

Foil-Gain

$$Foil-Gain(L,R) \equiv t(\log_2 \frac{p_1}{p_1+n_1} - \log_2 \frac{p_0}{p_0+n_0})$$

- Where p_0 is the number of positive bindings of rule R and n_0 is the number of negative bindings, p_1 is the number of positive bindings of rule R´, n_1 is the number of negative bindings of rule R´, and t is the number of positive bindings of rule R which are still covered by R´
- Reduction due to L in the total bits needed to encode the classification of all positive bindings of R.

Learning Recursive Rule Sets

- Just include the target in the list of Predicates

- Need test to avoid learning rules sets that produce infinite recursion

Summary of FOIL

- FOIL extension of CN2
- General-to-specific search adding new literals
- Literals may introduce new variables
- Foil-Gains used as evaluation function
- FOIL has been shown to successfully learn recursive rule sets
- To handle noisy data, some tradeoff between accuracy, coverage, and complexity tells it when to stop adding new literals
- FOIL also performs post-pruning

Induction as Inverted Deduction

- Induction is the inverse of deduction
- Given some data D and some partial background theory B, learning generates a hypothesis h that together with B explains D
- More precisely, if the training data is a set of examples of the form $<x_i,f(x_i)>$ where x_i denotes the ith training example and $f(x_i)$ denotes its target value.
- Then learning is the problem of discovering h such that
$$(\forall <x_i, f(x_i)> \in D)(B \wedge h \wedge x_i)\ entails\ f(x_i)$$

Inverted Deduction Example I

- Target concept is Child(u,v)
- Single positive example Child(Bob,Sharon) where instance is described by Male(Bob), Female(Sharon), and Father(Sharon,Bob)
- General background knowledge of Parent(u,v) ← Father(u,v)
- Two of the many hypothesis that satisfy

  ($B \wedge h \wedge x_i$) entails $f(x_i)$ are:

  h1: Child(u,v) ←Father(v,u) and

  h2: Child(u,v) ← Parent(v,u)

Inverted Deduction Example II

- New predicates which were not present in the initial description can be introduced into the hypothesis - constructive induction
- Well understood algorithms for automated deduction
- Inverses of these procedures can automate inductive generalization

Inverse Entailment Operators

$$O(B,D) = h \textit{ such that } (\forall < x_i, f(x_i) > \in D)(B \wedge h \wedge x_i) \textit{ entails } f(x_i)$$

- Usually many hs so use Minimum Description Length
- Incorporating background knowledge allows a more rich definition of when the hypothesis is said to fit the data
- Several practical difficulties:
  - Noisy training data
  - First order logic is so expressive that the search is intractable - restricted forms of expression or additional second-order knowledge
  - The complexity of the hypothesis space search increases as background knowledge is increased

Inverting Resolution

- Resolution rule - Robinson 65 - sound and complete
- This operator used in Cigol
- C=A v B and C_2 = B v D
- Any literal present in C but not in C_1 must be present in C_2
- The literal that occurs in C_1 but not in C must be the literal removed by the resolution rule and therefore its negation must occur in C_2
- C_2 = A v ¬D or C_2 = A v ¬D v B
- Not deterministic! - so prefer shorter clauses
- Cigol uses inverse resolution with sequential covering but with 1st order representations

Resolution

P v L
¬L v R

P v R

C_1: Pass v ¬KnowMaterial

C2: KnowMaterial v ¬Study

C: PassExam v ¬Study

Inverse Resolution

C_1: Pass v ¬KnowMaterial

C2: KnowMaterial v ¬Study

C: PassExam v ¬Study

First Order Resolution

- Substitutions
- θ = x/Bob, y/z, L=Father(x,Bill)
- $L\theta$ = Father(Bob,Bill)
- Unifying substitutions
- L_1=Father(x,y), L_2=Father(Bill,z), θ=x/Bill,z/y
- $L_1\theta = L_2\theta$=Father(Bill,y)
- $C=(C_1-L_1)\theta \cup (C_2-L_2)\theta$

Example

- C_1=White(x)←Swan(x) and
  C_2=Swan(Fred)
- L_1=¬Swan(x)
- L_2=Swan(Fred)
- θ=x/Fred
- L_1θ=¬L_2θ=¬Swan(Fred)
- C=White(Fred)

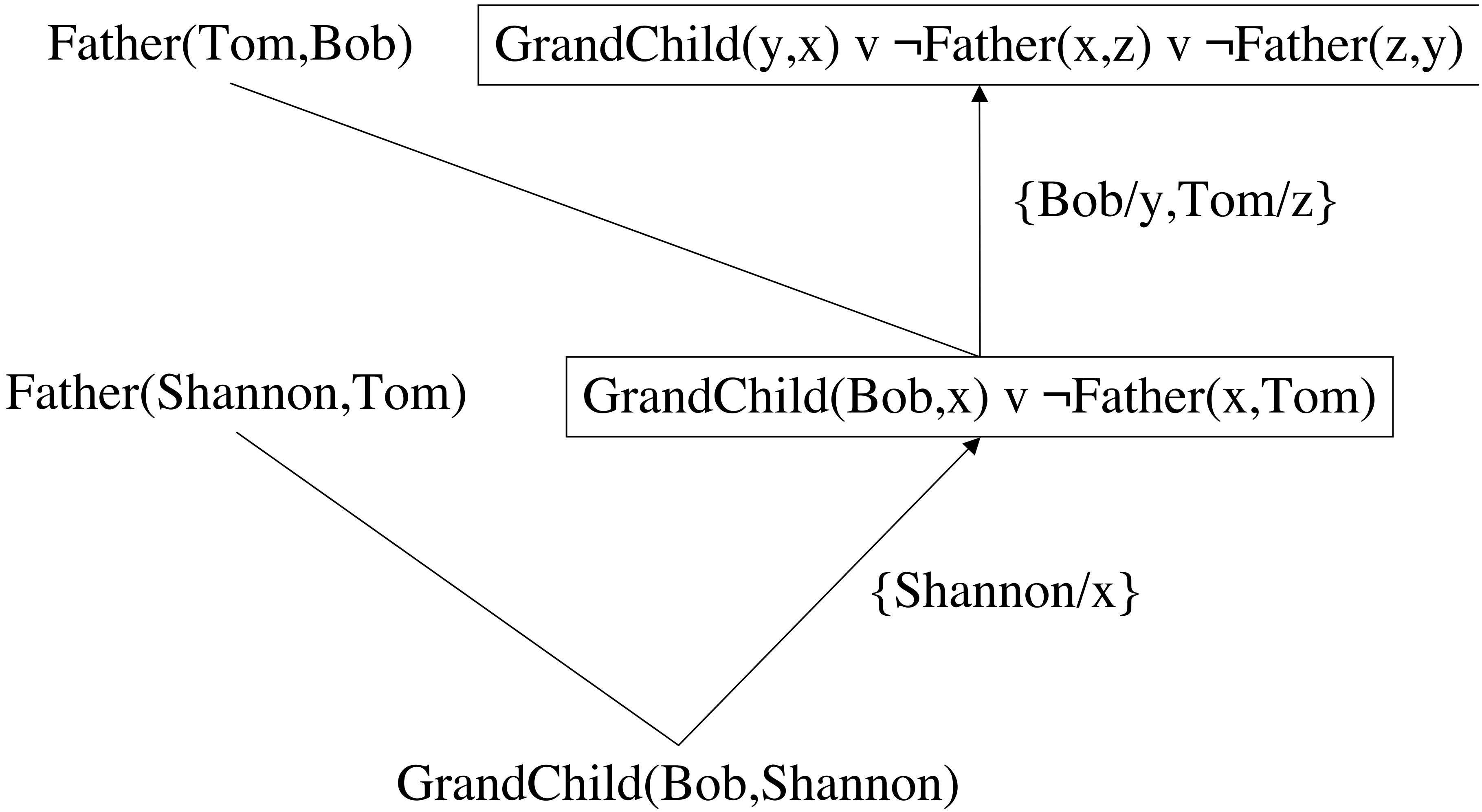
1st Order Resolution Rule

- Find a literal L_1 from clause C_1, literal L_2 from clause C_2, and substitution θ such that $L_1\theta = \neg L_2\theta$
- Form the resolvent C by including all literals from $C_1\theta$ and $C_2\theta$, except for $L_1\theta$ and $\neg L_2\theta$. More precisely, the set of literals occurring in the conclusion C is $C = (C_1 - \{L_1\})\theta \cup (C_2 - \{L_2\})\theta$

Inverting First Order resolution

- $C_2=(C-(C_1-L_1\theta_1)\theta_2^{-1}\cup\neg L_1\theta_1\theta_2^{-1}$
- Nondeterministic because of C1,θ_1,θ_2
- Grandchild(y,x)$\leftarrow$Father(x,z) ^ Father(z,y)

Inverse Example



Summary Inverse Resolution

- Only generates “good” hypothesis as opposed to generate and test
- So we would expect it to be more focused and efficient
- But hobbled because can only consider a small fraction of the data when generating a hypothesis at each step

Generalization, Subsumption, Entailment

- More general than - given two boolean functions $h_j(x)$ and $h_k(x)$ we say that $h_j \geq_g h_k$ if and only if $(\forall x) h_k(x) \rightarrow h_j(x)$
- θ-subsumption - Clause C_j is said to θ-subsume clause C_k if an only if there exists a substitution θ such that $C_j\theta \subseteq C_k$
- Entailment C_j entails C_k if and only if C_k follows deductively from C_j

Inverse Examples

- A θ-subsumes B but A is not more general than B
  - A: Mother(x,y)←Father(x,z) ^ Spouse(z,y)
  - B: Mother(x,Louise)←Father(x,Bob) ^ Spouse(Bob,y) ^ Female(x)
- A entails B but A does not θ-subsume B
  - A: Elephant(father_of(x))←Elephant(x)
  - B: Elephant(father_of(father_of(y)))←Elephant(y)

Progol

- Inverse entailment to generate the single most specific hypothesis
- Then general to specific search using this bound

1. Restricted language
2. Sequential Covering
    1. Inverse entail most specific hypothesis
    2. General to specific search

Summary

- Sequential covering learns disjunctive set of rules
- First-order rules with FOIL
- Inverse entailment