Informed search algorithms

Chapter 4, Sections 1–2, 4

Outline

- ♢ Best-first search
- ♢ A^* search
- ♢ Heuristics
- ♢ Hill-climbing
- ♢ Simulated annealing

Review: Tree search

function TREE-SEARCH(*problem*, *fringe*) **returns** a solution, or failure
  fringe ← INSERT(MAKE-NODE(INITIAL-STATE[*problem*]), *fringe*)
  loop do
    if *fringe* is empty **then return** failure
    node ← REMOVE-FRONT(*fringe*)
    if GOAL-TEST[*problem*] applied to STATE(*node*) succeeds **return** *node*
    fringe ← INSERTALL(EXPAND(*node*, *problem*), *fringe*)

A strategy is defined by picking the *order of node expansion*

Best-first search

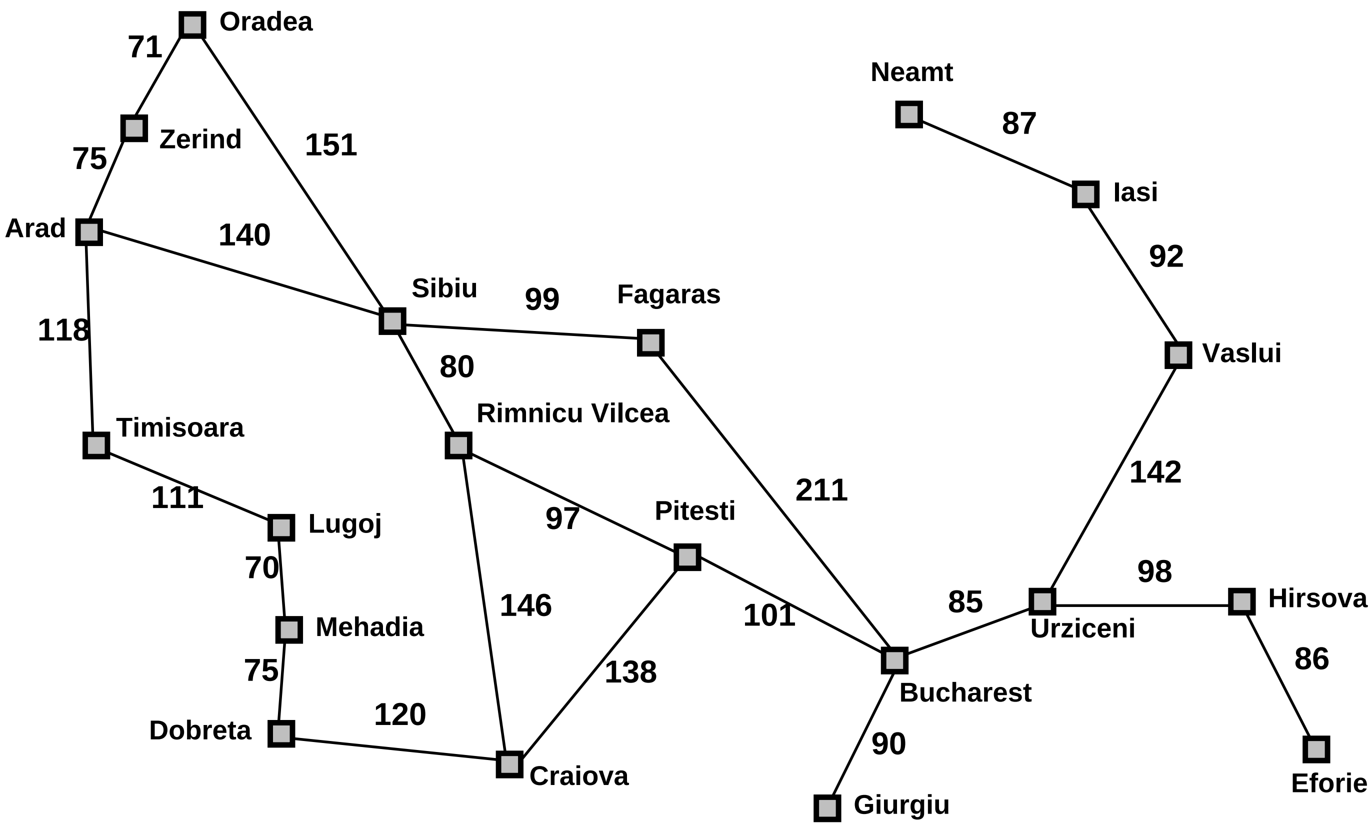
Idea: use an *evaluation function* for each node
– estimate of "desirability"

⇒ Expand most desirable unexpanded node

Implementation:
$fringe$ is a queue sorted in decreasing order of desirability

Special cases:
greedy search
A^* search

Romania with step costs in km



Straight−line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Greedy search

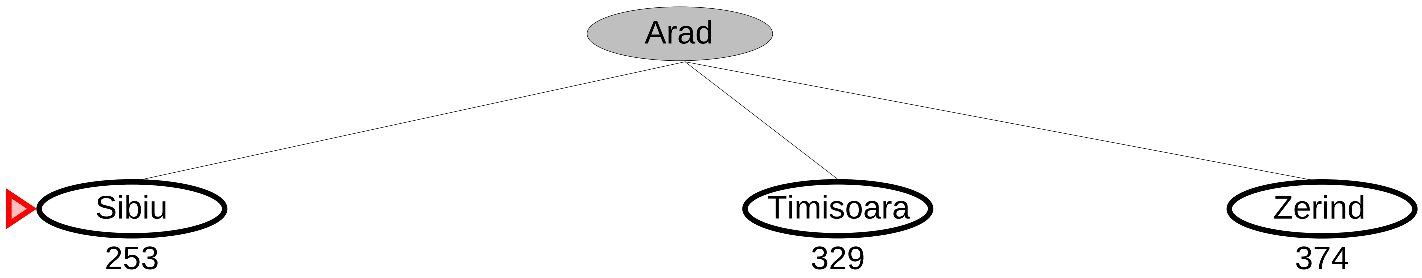
Evaluation function $h(n)$ (heuristic)
$= $ estimate of cost from n to the closest goal

E.g., $h_{\mathrm{SLD}}(n) =$ straight-line distance from n to Bucharest

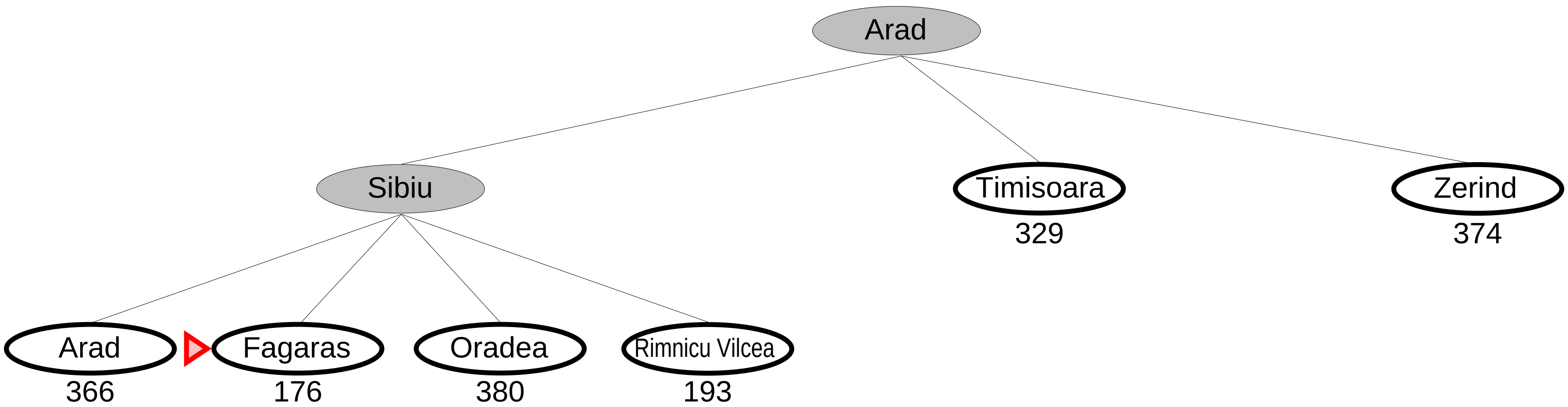
Greedy search expands the node that *appears* to be closest to goal

Greedy search example

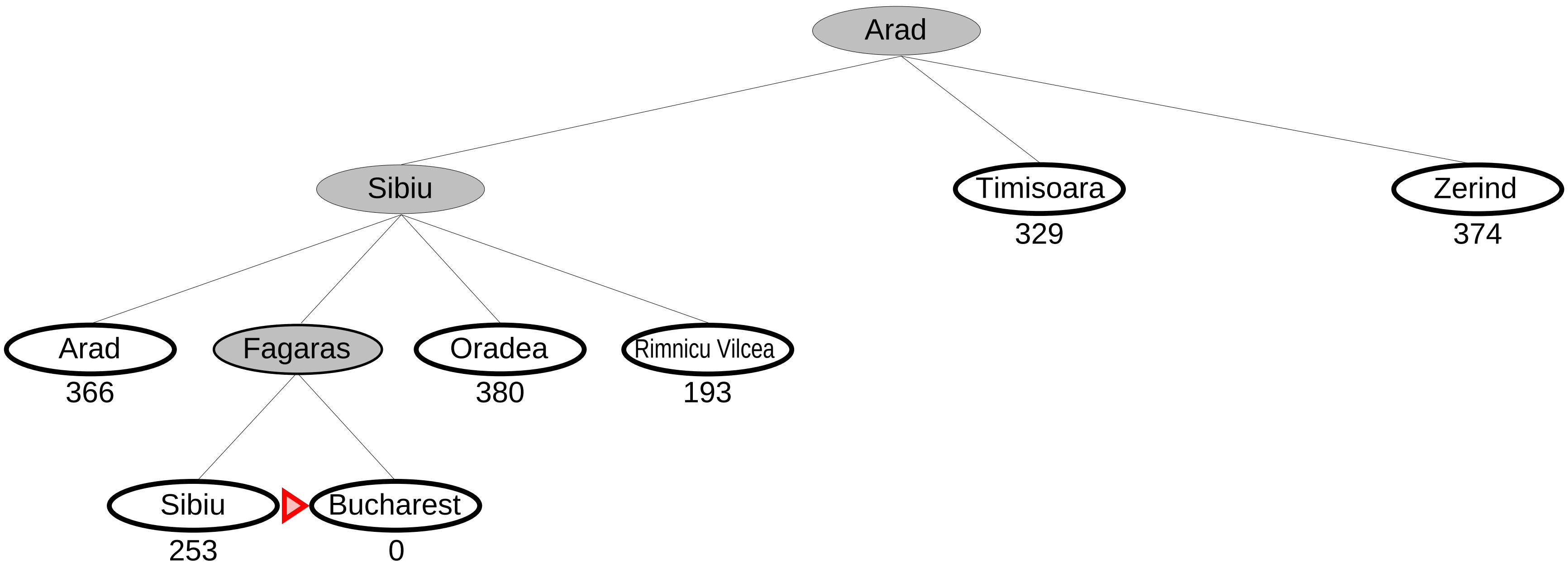
Greedy search example



Greedy search example



Greedy search example



Properties of greedy search

Complete??

Properties of greedy search

Complete?? No–can get stuck in loops, e.g., with Oradea as goal,

Iasi $\rightarrow$ Neamt $\rightarrow$ Iasi $\rightarrow$ Neamt $\rightarrow$

Complete in finite space with repeated-state checking

Time??

Properties of greedy search

Complete?? No–can get stuck in loops, e.g.,

Iasi → Neamt → Iasi → Neamt →

Complete in finite space with repeated-state checking

Time?? $O(b^m)$, but a good heuristic can give dramatic improvement

Space??

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Space?? $O(b^m)$—keeps all nodes in memory

Optimal??

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Complete in finite space with repeated-state checking

Time?? $O(b^m)$, but a good heuristic can give dramatic improvement

Space?? $O(b^m)$—keeps all nodes in memory

Optimal?? No

A* search

Idea: avoid expanding paths that are already expensive

Evaluation function $f(n) = g(n) + h(n)$

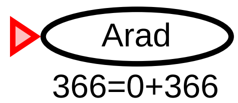
$g(n)$ = cost so far to reach n
$h(n)$ = estimated cost to goal from n
$f(n)$ = estimated total cost of path through n to goal

A* search uses an *admissible* heuristic
i.e., $h(n) \leq h^*(n)$ where $h^*(n)$ is the *true* cost from n.
(Also require $h(n) \geq 0$, so $h(G) = 0$ for any goal G.)

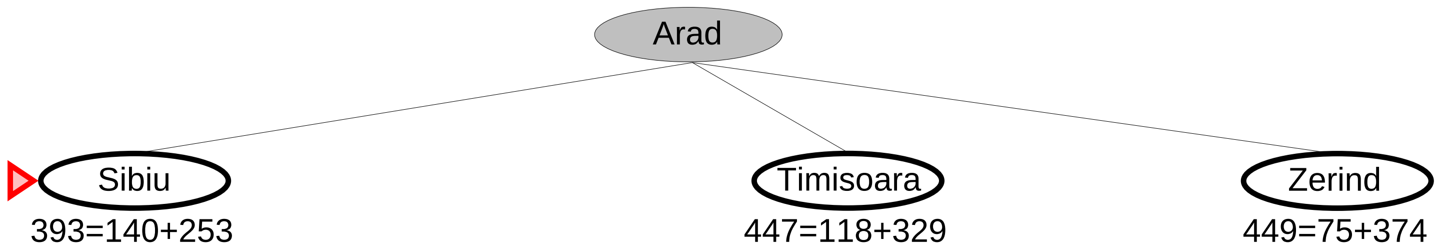
E.g., $h_{\text{SLD}}(n)$ never overestimates the actual road distance

Theorem: A* search is optimal

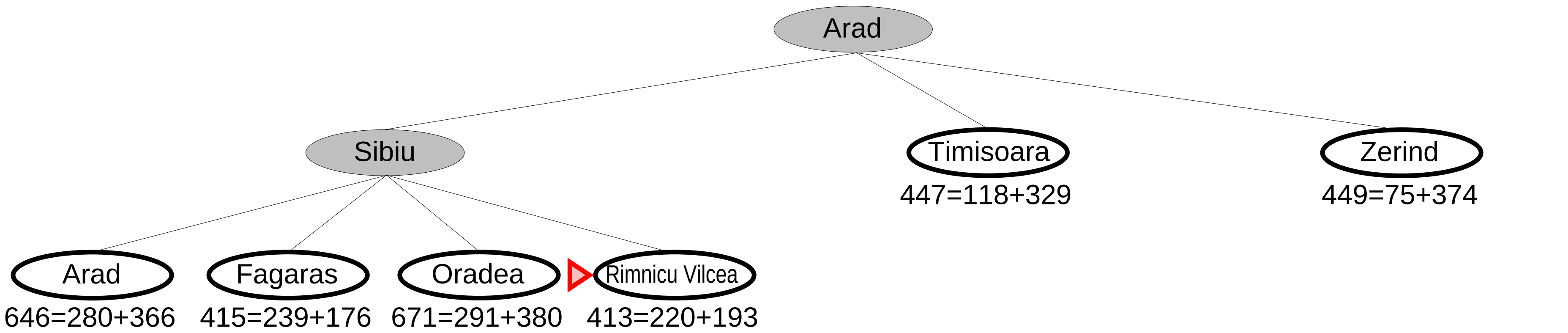
A* search example



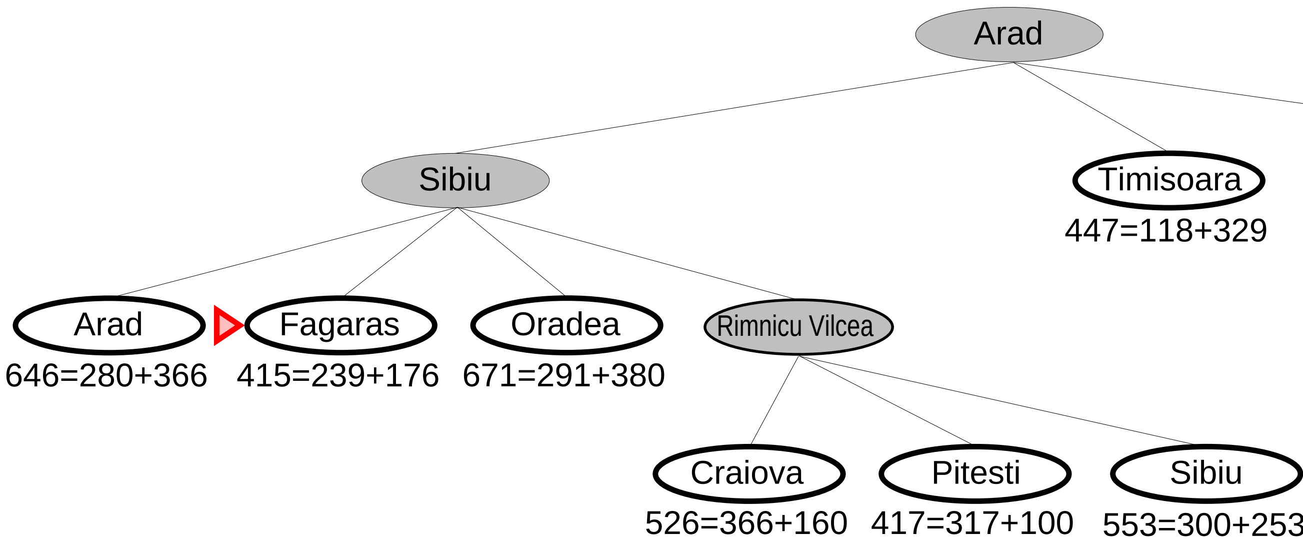
A* search example



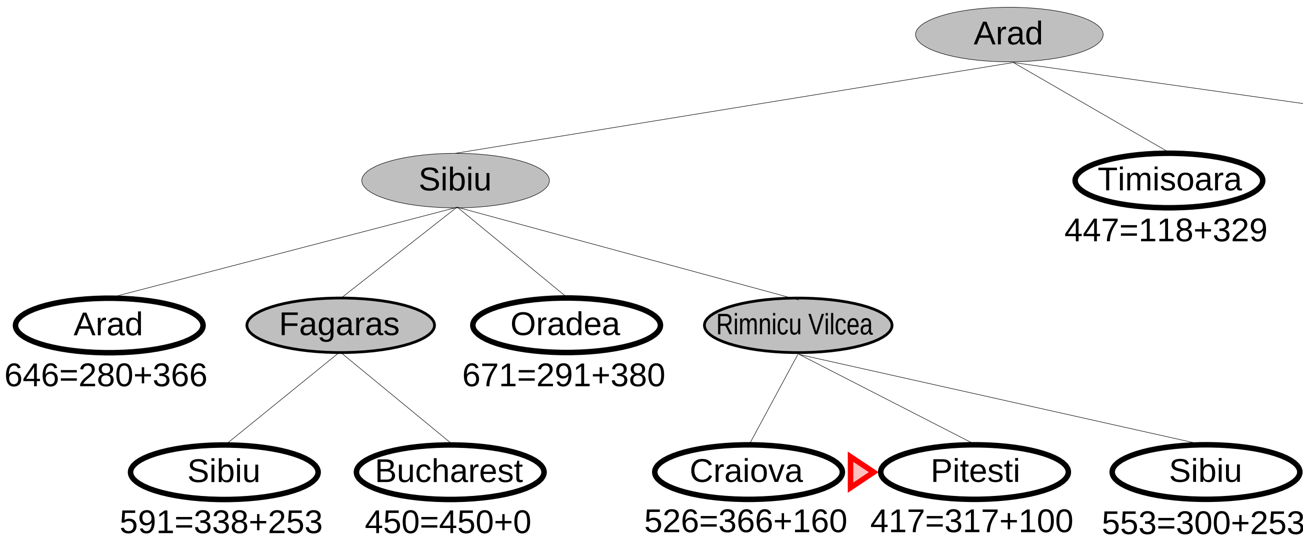
A* search example



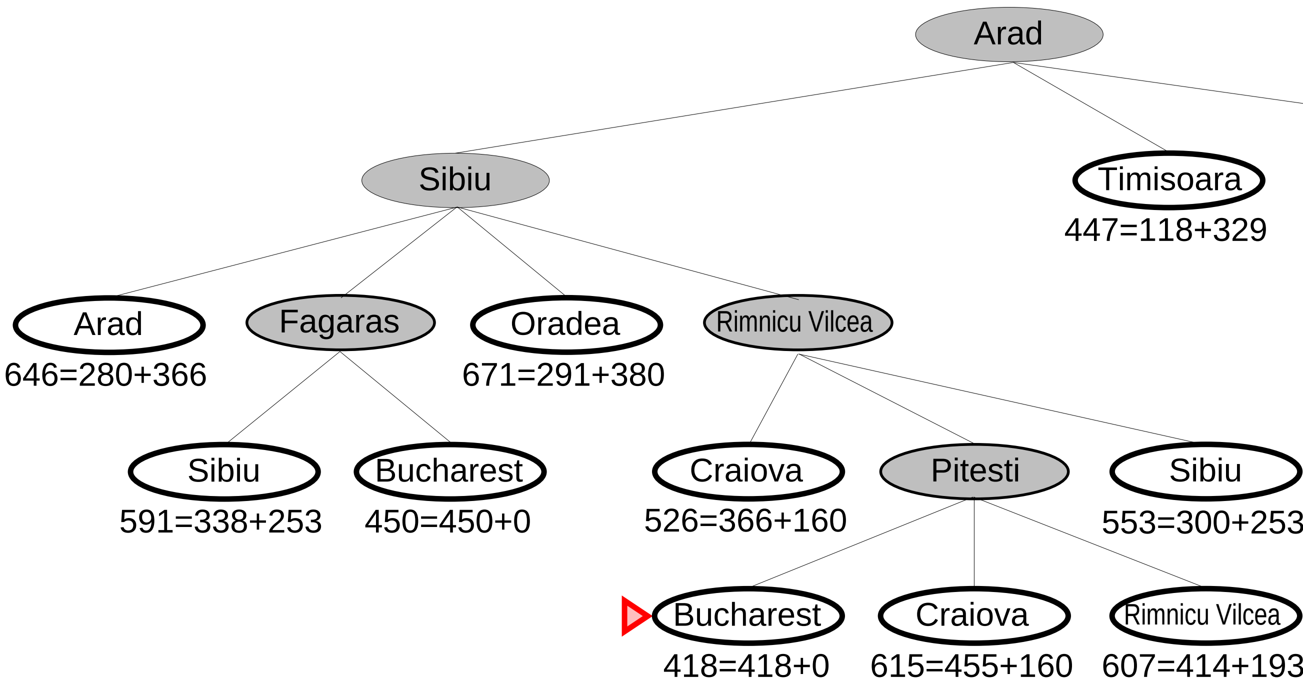
A* search example



A^* search example



A* search example

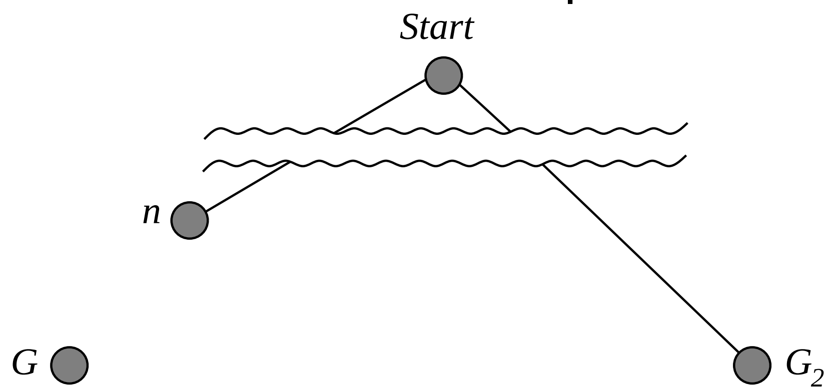


Types of Optimality

- Optimal Algorithm: guaranteed to find optimal solution.
- Optimally Efficient Algorithm: guaranteed not to expand any node that would not be expanded by a less informed optimal algorithm.

Optimality of A* (standard proof)

Suppose some suboptimal goal G_2 has been generated and is in the queue. Let n be an unexpanded node on a shortest path to an optimal goal G_1.



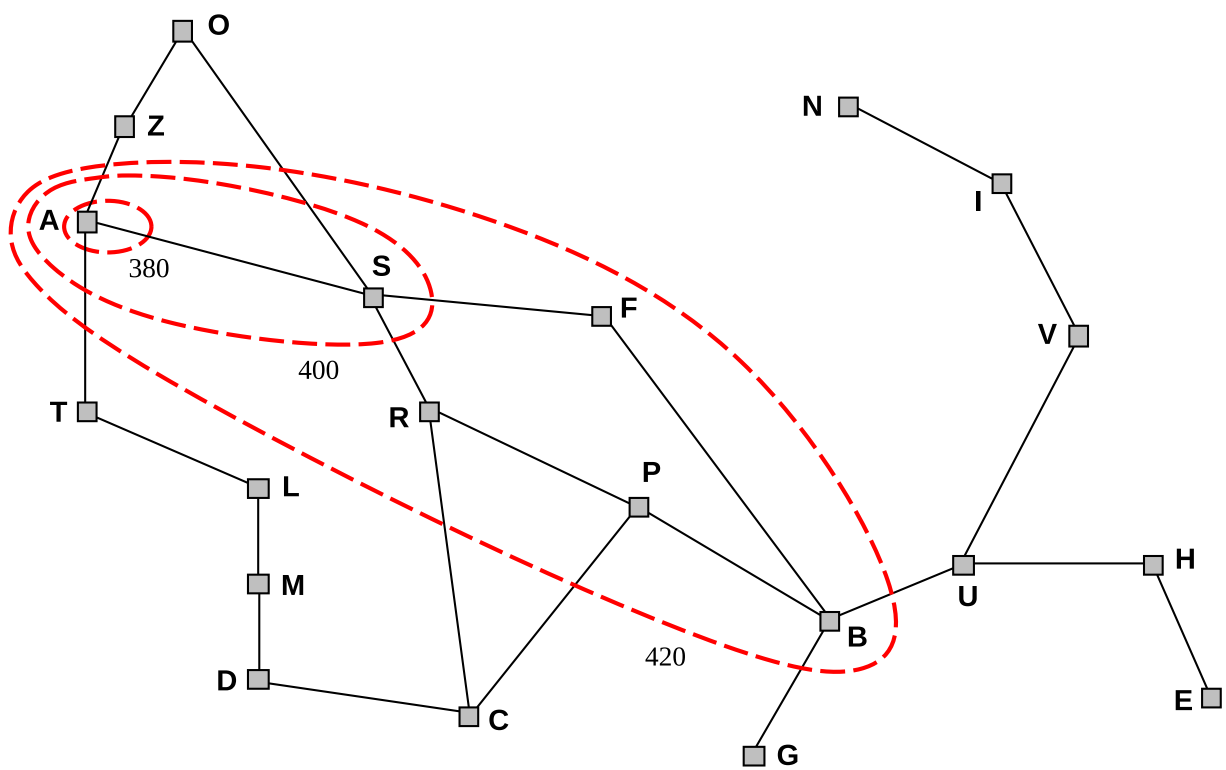
$$
\begin{aligned}
f(G_2) &= g(G_2) && \text{since } h(G_2) = 0 \\
&> g(G_1) && \text{since } G_2 \text{ is suboptimal} \\
&\geq f(n) && \text{since } h \text{ is admissible}
\end{aligned}
$$

Since $f(G_2) > f(n)$, A* will never select G_2 for expansion

Optimality of A* (more useful)

Lemma: A* expands nodes in order of increasing f value*

Gradually adds "f-contours" of nodes (cf. breadth-first adds layers)
Contour i has all nodes with $f = f_i$, where $f_i < f_{i+1}$



A* - Optimally Efficient

Informedness

A heuristic h_1 is less informed than heuristic h_2 if for all non-goal nodes n: $h_1(n) < h_2(n)$.

A* is Optimally Efficient

Proof:

Assume that h_1 is less informed than h_2 and that there exists a non-goal node n such h_2 expands n but h_1 does not. This means $f_{h1}(n) \geq f_{h2}(n)$.

Consider $f_{h1}(n) = g(n) + h_1(n)$ and $f_{h2}(n) = g(n) + h_2(n)$
Then $h_1(n) \geq h_2(n)$ but h_1 is less informed than h_2.

Therefore n cannot exist.

Properties of A*

Complete??

Properties of A*

Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time??

Properties of A*

Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time?? Exponential in [relative error in $h \times$ length of soln.]

Space??

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Space?? Keeps all nodes in memory

Optimal??

Properties of A*

Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time?? Exponential in [relative error in $h \times$ length of soln.]

Space?? Keeps all nodes in memory

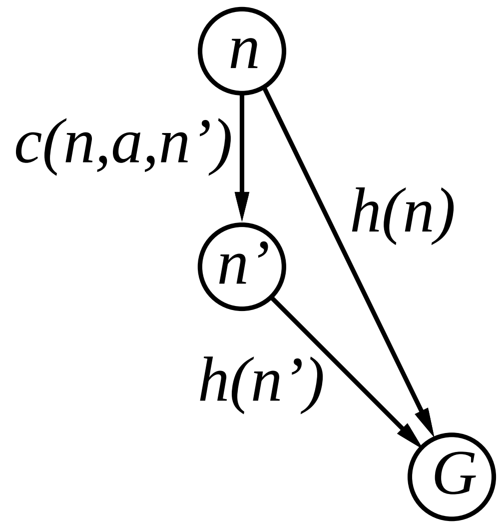
Optimal?? Yes—cannot expand f_{i+1} until f_i is finished

A* expands all nodes with $f(n) < C^*$
A* expands some nodes with $f(n) = C^*$
A* expands no nodes with $f(n) > C^*$

Proof of lemma: Consistency

A heuristic is *consistent* if

$$h(n) \leq c(n, a, n') + h(n')$$



If h is consistent, we have

$$\begin{aligned} f(n') &= g(n') + h(n') \\ &= g(n) + c(n, a, n') + h(n') \\ &\geq g(n) + h(n) \\ &= f(n) \end{aligned}$$

I.e., $f(n)$ is nondecreasing along any path.

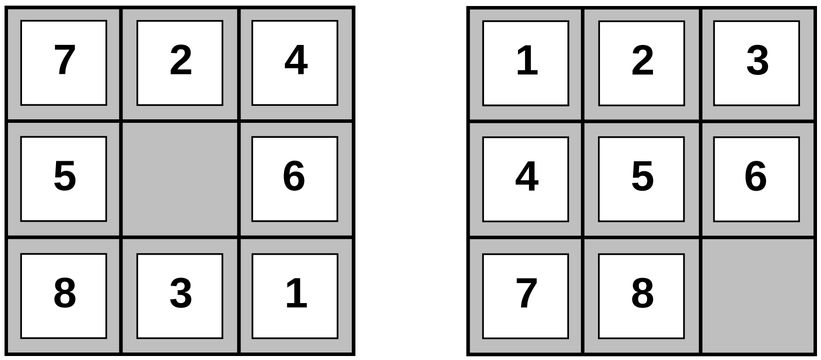
Consistency & Tree vs Graph Search

- When not worrying about duplicate states, don’t need to worry about consistency of heuristics.
- When worrying about duplicate states (e.g., graph searching) if heuristic is consistent then the first time you hit a state you have found the optimal path to it and you can throw away all the later paths to it.
- If the heuristic is not consistent then whenever you hit a path to an already generated state, you need to check whether the new path is shorter than the recorded path and if so then update the recorded information.

Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles
$h_2(n)$ = total Manhattan distance
(i.e., no. of squares from desired location of each tile)



Start State Goal State

$h_1(S) =$??
$h_2(S) =$??

Admissible heuristics

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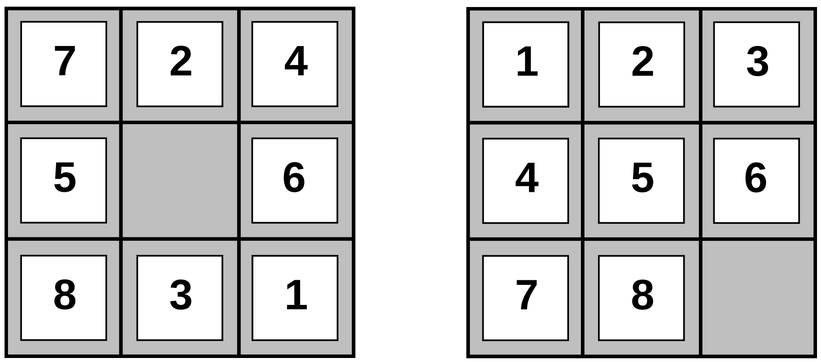

Start State **Goal State**

$h_1(S) =$?? 7
$h_2(S) =$?? 4+0+3+3+1+0+2+1 = 14

Dominance

If $h_2(n) \geq h_1(n)$ for all n (both admissible)
then h_2 *dominates* h_1 and is better for search

Typical search costs:

$d = 14$ IDS $=$ 3,473,941 nodes
A$^*(h_1) =$ 539 nodes
A$^*(h_2) =$ 113 nodes
$d = 24$ IDS $\approx$ 54,000,000,000 nodes
A$^*(h_1) =$ 39,135 nodes
A$^*(h_2) =$ 1,641 nodes

Relaxed problems

Admissible heuristics can be derived from the *exact* solution cost of a *relaxed* version of the problem

If the rules of the 8-puzzle are relaxed so that a tile can move *anywhere*, then $h_1(n)$ gives the shortest solution

If the rules are relaxed so that a tile can move to *any adjacent square*, then $h_2(n)$ gives the shortest solution

Key point: the optimal solution cost of a relaxed problem
is no greater than the optimal solution cost of the real problem