Linking Algebra and Geometry – Human Computer Interaction and Mathematical Pedagogy

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ABSTRACT

This paper is inspired by M. Hohenwarter's analysis of GeoGebra. Hohenwarter makes very strong claims regarding the synthesis of geometry and algebra. This literature review paper attempts to redress fundamental flaws in Hohenwarter's understanding of algebra pedagogy in the formation of a seemingly effective human computer interaction model.

In an analysis of the weak first principals of Hohenwarter's paper, it will be shown that his human computer interaction model may actually be detrimental to student comprehension. It is hoped that this exploration will serve as a warning to HCI developers' domain analysis. Hohenwarter's inability to appreciate first principals of his target domain has arguably led to a potentially harmful pedagogical tool. This human computer interface – incorporating the synthesis of algebra and geometry – will be examined in light of established pedagogical practice.

Apart from GeoGebra, variant geometry and algebra visualization software will be documented and analyzed, as possible, in regards to its benefit to student learning. These alternatives will be reviewed in terms of their ability to adhere to established pedagogical standards.

Finally an analysis of effective and tested geometric and algebraic visualization software will lead to an effective proposed model of human computer interaction.

Author Keywords

Human Information Processing; Pedagogy; HCI;

ACM Classification Keywords

H.1.2 . Human information processing

INTRODUCTION

Algebra is recognized by the New Oxford American Dictionary as *"The part of mathematics in which letters and other general symbols are used to represent numbers and quantities in formulae and equations"*. The late Middle English origins of the word describe algebra as "*the science of restoring what is missing and equating like with like*". This is reiterated in the etymological origin of the word with originally denoted a "*reunion of broken parts or bones setting*" [14]. It is into this already complete symbolic formalization that new human computer interaction techniques are being developed in order to complement mathematical ontology.

The question of whether or not these techniques will successfully improve learning practices will be gauged in comparison to their pedagogical foundations and claims. Hohenwarter, in his recent paper on the synergy of geometry and algebra, introduces a new tool in the array of mathematical software appliances [10].

In addition to GeoGebra, work has progressed, towards ever improving geometric and algebraic visualization software. Unfortunately the world of mathematical software appears to be underfunded [1]. Secondarily, the development of mathematical applications involves the embodiment of abstract principals. The creation of abstract visualization results in complex software engineering architectures [1]. These two mitigating factors produce a protracted field of available mathematical software. Apart from classic CAS systems, such as Mathematica, Maple, etc, it is important to look at some of the alternatives to GeoGebra in an attempt to recognize an empirically successful geometric or algebraically visual software application.

THE SUCCESS OF GEOGEBRA?

Hohenwarter presents GeoGebra as an attempt to unite geometry and algebra in the classroom. The function of current technology can be visualized as a dichotomy of algebraic and geometric tools. Computer Algebra Systems (CASs) exist to process algebraic equations in a manner that is highly accelerated when compared to traditional pen and paper methodologies. Similar to advanced graphing calculators, CASs can produce visualizations of the equations that they manipulate [5].

At the opposing end of the mathematical learning tools spectrum is software which allows for direct manipulation of geometric objects [13, 16, 19]. The synthesis of CAS and geometric software is aimed at allowing manipulation of algebraic equations via geometric mechanisms. The express purpose, according to Hohenwarter, is not geometric understanding, but rather algebraic, as the student is able to interact with the geometric representations of algebraic principles.

The technological achievement produced by this synthesis is an appliance that allows for seamless transitions between two mathematical domains. While CAS software allows for transitions from algebraic symbolic formalization to geometric representation and geometric software allows for direct manipulation of geometric objects, GeoGebra is one of few software packages that allow reflections of geometric manipulations into the algebraic domain. In effect this software package is not a new interface but a hybridization of existing mathematical software.

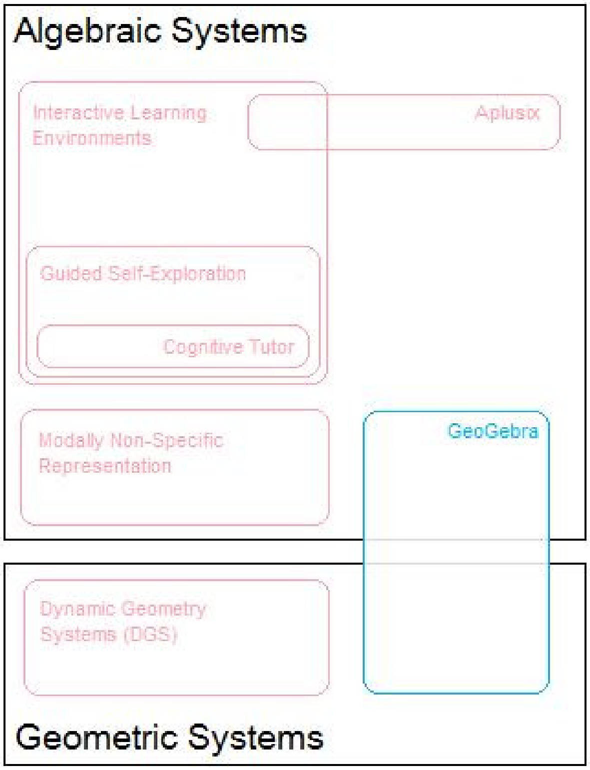


Figure 1 Domain of Pedagogical Tools

Why GeoGebra Fails

While GeoGebra illustrates an astounding synergy between existing applications, it is potentially an abject failure when compared to its proposed objectives. Hohenwarter quotes Atiyah when considering his problem domain: *"Algebra is concerned with manipulation in time, and geometry is concerned with space. These are two orthogonal, aspects of the world, and they represent two different points of view in mathematics. Thus the argument or dialogue between mathematicians in the past about the relative importance of geometry and algebra represents something very, very fundamental."* [10] Hohenwarter quotes Atiyah again in stating that: "*when you pass over into algebraic calculation, essentially you stop thinking; you stop thinking geometrically, you stop thinking about the meaning*" [10]. From this presentation we can see that Atiyah is critical of algebraic reasoning. Furthermore, Hohenwarter attempts to illustrate geometry and algebra as spectrum, in opposition to a dichotomy of two separate mathematical domains.

What this rationalization neglects is two primary facts regarding these mathematically divergent spheres. Primarily, classic algebraic education focuses on understanding symbolic formalizations, not general problem solving techniques [15]. In conflict with Hohenwarter, over a thousand years of mathematical education in algebraic principles has utilized geometry as a proof mechanism. In short, established pedagogical practices have utilized geometry as a proof technique not an algebraic visualization mechanism. Geometry as a domain of algebraic visualization is recognized as an application of "constructionism". Constructionism was developed as a pedagogical mechanism in the 1980 and has been questioned by leading HCI developers as a valid mechanism for mathematical education [11].

Secondarily, the use of constructionism principles in human computer interaction software has lead to the use of an educational tool that has been empirical shown to, at best, produce nominal results. Yerushalmy in a recent study on the effects of geometric reasoning when applied to algebra found that the use of an alternate domain for algebraic manipulation actually eroded the students' ability to understand algebraic symbolism [20]. While their test scores remained nominal, their understanding of algebraic symbolism actually decreased.

In short, traditional pedagogy and empirical results have illustrated that the synthesis of these two mathematical domains for algebraic visualization is counterproductive to the development of sound mathematical formalizations.

The Benefits of GeoGebra

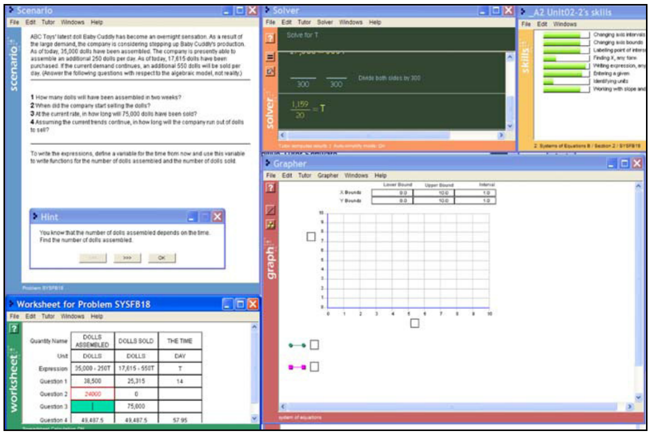
The synergy of two disparate mathematical domains may not produce an effective pedagogical tool in the hands of students (as proposed by Hohenwarter) however it may be valuable at enforcing existing pedagogical techniques related to geometric proof techniques. With the formulation of effective examples by teaching staff, the geometric manipulations in GeoGebra may produce enriched proof demonstrations in the hands of informed teaching staff [6]. Of course the relegation of GeoGebra to a tool solely for the production of object lessons, by teaching staff, undermines the value of this software package in that its dynamic visualizations become less relevant without free form student interaction.

ALTERNATE GEOMETRIC AND ALGEBRAIC VISUALIZATION SOFTWARE

Beth Bos illustrates that one of the most effective mechanisms for mathematical education is the introduction of patterns that can improve cognitive fidelity through pattern recognition [2]. While this might superficially appear to correspond with Hohenwarter's thesis, the following examples should illustrate the difference between guided explorations towards proof theory, in opposition to the transference of problems to another domain which may erode student understanding.

Cognitive Tutor (CT) by Carnegie Melon University

Ken Koedinger, John Anderson and Steve Ritter of Carnegie Melon University created Cognitive Tutor (CT) in 1983. Cognitive Tutor is an algebraic tutoring software package that anticipates student's mistakes. John Anderson, in early research determined that the majority of student errors can be categorized into finite categories. In this fashion, CT creates an environment that can produce learning outcomes based upon student errors [1].



ABC Toys' latest doll Baby Cuddly has become an overnight sensation. As a result of the large demand, the company is considering stepping up Baby Cuddly's production. As of today, 35,000 dolls have been assembled. The company is presently able to assemble an additional 250 dolls per day. As of today, 17,615 dolls have been purchased. If the current demand continues, an additional 550 dolls will be sold per day. (Answer the following questions with respect to the algebraic model, not reality.)

1 How many dolls will have been assembled in two weeks?
2 When did the company start selling the dolls?
3 At the current rate, in how long will 75,000 dolls have been sold?
4 Assuming the current trends continue, in how long will the company run out of dolls to sell?

To write the expressions, define a variable for the time from now and use this variable to write functions for the number of dolls assembled and the number of dolls sold.

You know that the number of dolls assembled depends on the time. Find the number of dolls assembled.

Solve for T

Divide both sides by 300

$\frac{1,159}{20} = T$

Quantity Name	DOLLS ASSEMBLED	DOLLS SOLD	THE TIME
Unit	DOLLS	DOLLS	DAY
Expression	35,000 + 250T	17,615 + 550T	T
Question 1	38,500	25,315	14
Question 2	24000	0	
Question 3		75,000	
Question 4	49,487.5	49,487.5	57.95

Figure 2 Cognitive Tutor

The final purpose of this package is the introduction of mathematical formalizations via student self discovery. A similar package was developed by the United States Army in order to teach writing skills [4]. Both the militaries writing package (R-WISE) and CT remain within their educational domains while utilizing self discovery in order to reinforce core principals. Both R-WISE and CT have been trialed extensively in educational settings. Results based on studies of CT show that it either improves or creates nominal effect upon student learning. Further development of both packages continues as a result of robust financial endowments [1, 4].

General Dynamic Geometry Software

As Metaxas and Karagainnidou state *"visualization of mathematical relationships enables students to formulate conjectures as well as to search for mathematical arguments to support these conjectures.* However it should be noted that this development of conjectures and mathematical arguments takes place strictly within the bounds of dynamic geometric software according to Metaxas and Karagainnidou's paper [13].

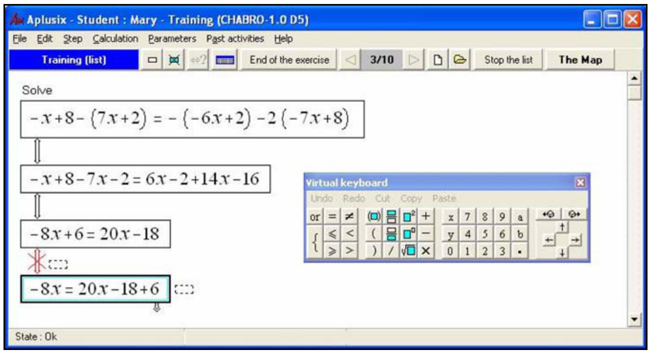
Through the use of software such as Sketchpad and Autograph, it is shown that Dynamic Geometric Software (DGS) is capable of illustrating advanced mathematical arguments without intersecting variant mathematical domains, such as algebra. Weaver and Quinn show that the use of Sketchpad allows for powerful exploration of mathematical principles. Accordingly, it can afford simplifications in the understanding of geometric principles. Similar to the results illustrated in Anderson's analysis of Cognitive Tutor, this ability to enable self discovery, towards mathematical established principles, produces increased cognitive fidelity. It should, however, be noted that Weaver and Quinn note that Sketchpad is an insufficient pedagogical tool, without teacher input and the formulation of sufficient "*descriptions and justification*" [19].

Regarding Dynamic Geometric software Weaver and Quinn demonstrate qualitatively that software such as Sketchpad is capable of developing exploration via guided experimentation with geometric examples and geometric principles.

It can be seen from illustrations of Sketchpad, that robust manipulation of geometric relations are possible via experimentation with the software. Again, referring to Weaver and Quinn, examples of experimentation with polygons, circles and triangles are presented. The underlying geometric and mathematical formalisms and principles are evident from the robust and interactive framework that Sketchpad offers [19].

Aplusix

The developers of Aplusix recognize that the didactic principles garnered from pen and paper interaction are exceptionally different from current CAS systems. While traditional CAS systems attempt to incorporate menu systems, populated with further menus and buttons, Aplusix attempts to incorporate principles found in pen and paper mathematical exploration.



Solve

$-x+8-(7x+2) = -(-6x+2) -2(-7x+8)$

$-x+8-7x-2 = 6x-2+14x-16$

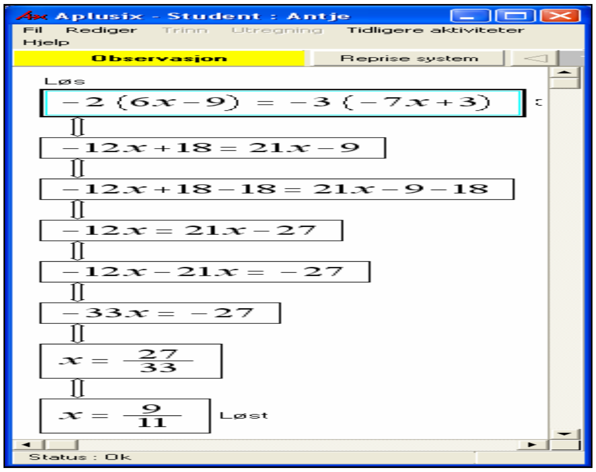
$-8x+6 = 20x-18$

$-8x = 20x-18+6$

Figure 3 Aplusix

Aplusix remains completely within the domain of algebra. However, the approach that Aplusix takes towards improved interaction is via a work flow implementation that mimics current pedagogical practices as opposed to innovating methods outside of current teaching best practice. Due to Aplusix's purposed alignment of classical algebraic formulation, experimentation was able to illustrate improvements in student pen and paper learning practices. Therefore, the synthesis of human computer interaction closer, as opposed to further, from accepted pen and paper best practice didactic principals allowed for demonstratively improved student understanding [8].

Experimental results illustrate that Aplusix was able to improve student understanding of symbolic formalism [8]. Where traditional CASs obscure symbolic formalism, Aplusix is able to bring symbolic principle to the fore, where other CAS software packages eschew symbolic principals by inventing new formalisms and hiding traditional modes of algebraic interaction through didactic principals that conform to computer affordances, not pedagogical principles.



$$-2(6x-9) = -3(-7x+3)$$
$$-12x+18 = 21x-9$$
$$-12x+18-18 = 21x-9-18$$
$$-12x = 21x-27$$
$$-12x-21x = -27$$
$$-33x = -27$$
$$x = \frac{27}{33}$$
$$x = \frac{9}{11}$$

Figure 4 Equivalence Relationship in Aplusix

Figure 4 illustrates an example of the formation of algebraic representations around balanced equivalence relations. The presentation of equivalence relationships can be eschewed by traditional CAS software while Aplusix clearly illustrates a student workflow that mimics traditional pen and paper interaction.

Aplusix's direct consideration of pedagogical practices appears to have created an educational tool that enforces didactic principles readily illustrated with direct manipulation of symbolic formalization. Examples of concepts empirically tested amongst students include equivalence relations, negation, arithmetic knowledge of zero and resolution of complex equations

Modally Non-Specific Algebraic Representation

Tatiana Everinova preformed an analysis on the translation of algebraic symbolism to a non-visual tactile domain. The advantage of Everinova's analysis is that this algebraic translation was not an ideological mapping; it did not translate algebraic principles into a domain of mathematical understanding that was already weighted with its own mathematical theory [7]. The use of tactile representations of algebraic symbolism illustrates the suitability of translating symbolic formalisms.

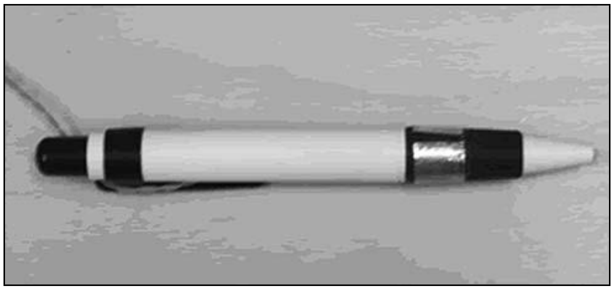
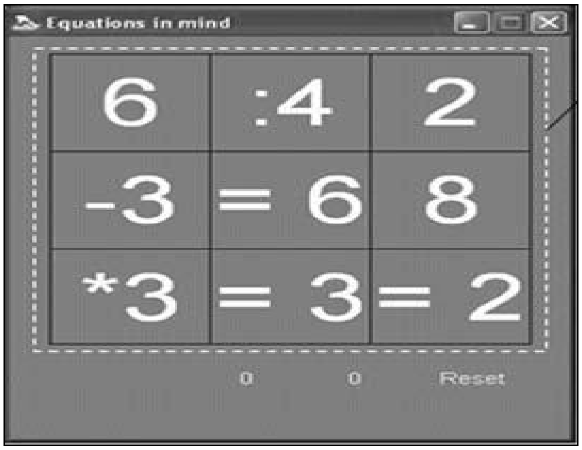


Figure 5 Elctro-Tactile Pen Device



6	:4	2
-3	= 6	8
*3	= 3	= 2

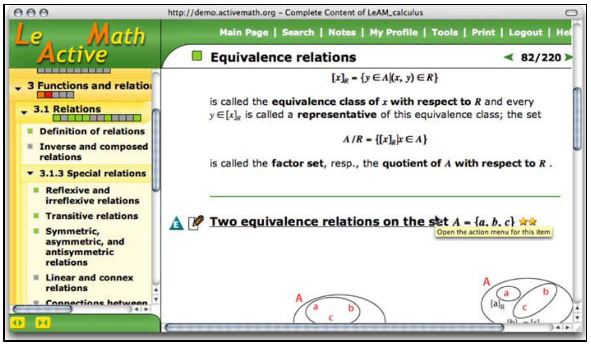
Figure 6 Algebraic Spatial Orientation Software

Everinova employs the use of a pressure sensitive OLED screen and a vibrating electro-tactile selection device. By providing shocks and vibration, the selection device is capable of presenting feedback to spatially oriented selection that the user makes upon the OLED screen. In this manner, algebraic equations can be created by a tutor and explored/solved by students who wish to interact with the algebraic symbolism in a spatial electro-tactile domain. The findings of Everinova's study were that the error rate amongst users of the system was high enough to reduce any indications of a successful algebraic translation. It should however be noticed that Everinova utilized a game-based testing format. Beth Bos has specifically stated that game-based mathematical fidelity is surprisingly low [2]. As such the failure of Everinova to translate symbolic formalism into spatial/tactile representations may be a failure to construct helpful experimental conditions as opposed to a failure of first principles

Interactive Learning Environments: Algebra Based Mathematical Exploration Software

Interactive Learning Environments (ILEs) are simply integrated mathematical exploration tools. Similar to GeoGebra, they provide exploration of mathematical concepts in a manner that may be tedious compared to pen and paper calculations [17].

ActiveMath is an example of an ILE. It incorporates various visualization techniques. It is web-based and most importantly, like Carnegie Melon Universities CT software, it anticipates and suggests student pathways toward a solution or proof.



Equivalence relations

$$[x]_R = \{y \in A | (x, y) \in R\}$$

is called the **equivalence class of x with respect to R** and every $y \in [x]_R$ is called a **representative** of this equivalence class; the set

$$A / R = \{[x]_R | x \in A\}$$

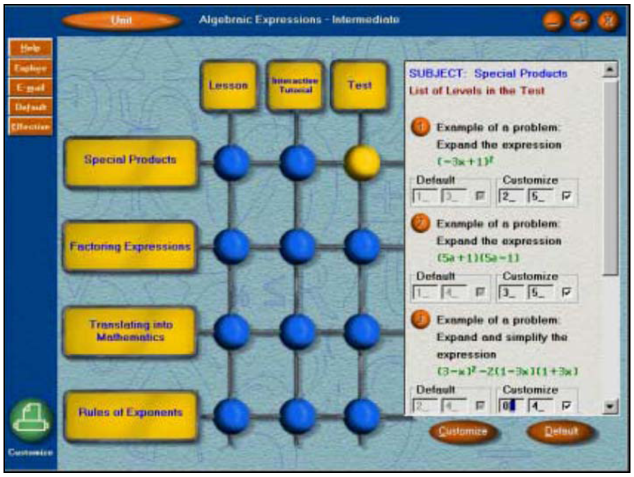
is called the **factor set**, resp., the **quotient of A with respect to R** .

Two equivalence relations on the set $A = \{a, b, c\}$

Figure 7 Active Math

An elementary software package is AnimalWatch, an ILE that attempts to translate arithmetic and algebraic principals into the domain of biology.

MathTeacher is an algebraic free-from exploration software package that allows for complete autonomy of student interaction within the algebraic domain. Again similar to ActiveMath and CT, MathTeacher provides guided feedback.



SUBJECT: Special Products

List of Levels in the Test

1. Example of a problem: Expand the expression $(-3x+1)^2$

Default 1 3 ☑ Customize 2 5 ☑

2. Example of a problem: Expand the expression $(5a+1)(5a-1)$

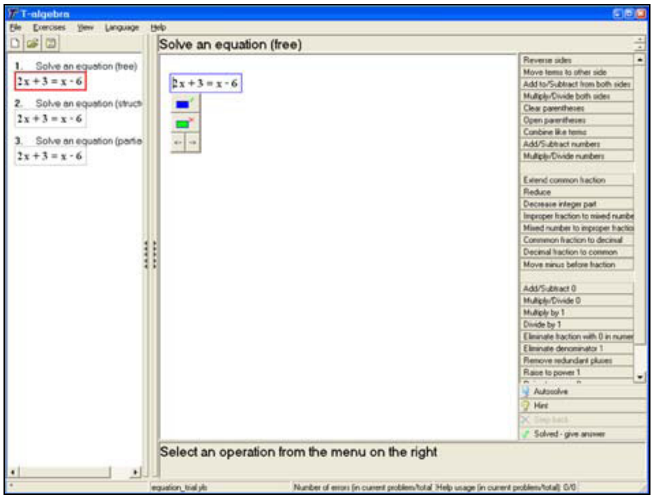
Default 1 4 ☑ Customize 3 5 ☑

3. Example of a problem: Expand and simplify the expression $(3-x)^2-2(1-3x)(1+3x)$

Default 2 4 ☑ Customize 0 4 ☑

Figure 8 Math Teacher

MathXpert allows for the creation of solution pathways. While an algebraic tool, it stresses the necessary steps required to create mathematical solutions. While other ILEs may attempt to acclimatize students to the necessary algebraic formalization, MathXpert attempts to enforce problem solving techniques or "development of computation". T-algebra is a similar ILE which stresses the formation of computations over symbolic formalism. AriLab2 extends the computational coaching of MathXpert and T-algebra through the use of lesson construction parameters, peer-to-peer collaboration and messaging.



1. Solve an equation (free)
$2x+3=x-6$

2. Solve an equation (struct
$2x+3=x-6$

3. Solve an equation (partie
$2x+3=x-6$

Solve an equation (free)

$2x+3=x-6$

Select an operation from the menu on the right

equation_trial.ylb Number of errors (in current problem/total Help usage (in current problem/total) 0/0

Figure 9 T-algebra

Finally E-slate, while not technically a complete ILE, it is a toolkit available for the creation of mathematical exploration software. The fact that E-slate is based on the Logo programming language illustrates that this solution is an obvious extension of constructionist pedagogy principles as outlined by Seymour Papert [3]. In this manner, while it is an incomplete environment, it is a prime example of the desire to produce mathematical tools based upon constructionism principles [17]. "Half-Baked" modules are available out-of-the-box in order to promote the creation of open ended translations of algebraic principles into other domains.

While ILEs do not represent a direct synthesis of geometric and algebraic concepts, they do – to varying degrees – represent the embodiment of various constructionist, computational, modally non-specific and pedagogical principles in the field of mathematics education.

SUCESSES IN GEOMETRIC AND ALGEBRAIC VISUALISATION SOFTWARE

Visualization of algebraic concepts within human computer interaction can be formally divided into discrete categories. ILEs, GeoGebra and development environments such as E-Slate are direct embodiments of Seymour Papert's constructionism pedagogy principles.

Dynamic Geometry Software illustrates an incarnation of classical geometric pedagogy. Geometry has, since ancient times, been a tool of proofs illustration. In this manner DGS tools leverage the success of thousands of years of mathematics pedagogy, by presenting concepts from within the domain of geometry. It should be noted that classic geometric proof techniques, which illustrate algebraic principles, have not required the synthesis of algebraic manipulation.

Everinova's experimentation with spatial and tactile representations of algebraic concepts illustrates a desire to translate algebraic formalizations into a domain that is accessible to students with visual acuity problems. It does not attempt to imbue algebraic concepts with new meaning or depth in its new spatial/tactile domain; it is an attempt at a direct translation, not a transformation.

Cognitive Tutor and various ILEs (ActiveMath, MathTeacher, MathXpert) allow for a solution pathway. It has been empirically demonstrated that guided self-exploration can contribute to student comprehension [1, 2, 5, 13, 17]. These techniques have been employed in non-mathematical fields, such as written English teaching (R-WISE). CTs success has even garnered substantial financial backing as it demonstrates results, in certain test situations, above average [1]. The classic employment of geographic illustrations as a guideline to proof techniques may be seen as an analogy to these guided algebraic instructions.

CONCLUSION

Unfortunately the assertion that constructionist principles are valuable to algebraic pedagogy is contentious at best [11]. Constructionist principles have only been formulated in the last 30 years by Seymour Papert. Hohenwarter states that his presentation of GeoGebra is an exploration of application affordances [10]. Hohenwarter explores the synthesis of Geometry and Algebra as a tool to "*enhance the teaching of mathematics*". A synthesis of Geometry and Algebra, two fields that have occupied different didactic domains for the last few millennia, is an evident embodiment of Papert principles [15]. By elevating affordances to the level of pedagogy, Hohenwarter has illustrated how the ascension of human computer interfaces can damaged or disregarded sound pedagogical practices.

Cleborne D. Maddux, editor of Computers in the Schools, states that "*Two of the most common ideas encountered in current journal articles are (a) the notion that we should base education on the psychology of constructivism, and (b) the contention that learning is situated in social experience. These ideas are related and occur so frequently and so often without reference to empirical studies that they appear to be sacrosanct beliefs that have become part of the accepted subculture of our field, and that are seldom, if ever questioned.*" [11] Secondly, Michal Yerushalmy in the Journal for Research in Mathematics Education states, empirically, that tools that migrate learning practices from algebra towards geometry actually erode students understanding of symbolic formalizations [20]. As the very definition of algebra is reasoning in symbolic formulation, it must be considered that constructionist tools that synthesize mathematic domains and degrade symbolic formalism are potentially destructive to algebraic reasoning, even if they are able to promote problem solving abilities and general reasoning.

In contrast to tools that dictate pedagogy via software affordances, Cognitive Tutor, ActiveMath, MathTeacher and MathXpert encapsulate sound pedagogical principals. Tools that encapsulate guided self-exploration, sound mathematical pedagogy and the extension of empirical results stand in direct opposition to the philosophy that produces pedagogy from the affordances allowed in CAS and geometric software. In an environment where advances in mathematical teaching software have failed to produce convincing results, it could be argued that future developments would be wise to consider and embody successful pedagogical techniques [12, 18, 20].

While efforts have been made to create viable teaching platforms, GeoGebra is a confusion of human computer interaction affordances with sound pedagogy [9]. Mathematics is a discipline that covers millennia of proven results, the study of pedagogy likewise, encapsulates centuries of debate and reasoning. Unfortunately these findings have been shunned in order to accommodate novel human computer interface research. Until human computer interface experiments, such as GeoGebra, can produce empirical results, they should be seen as a failure of HCI to distinguish innovation from novelty.

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