THE UNIVERSITY OF AUCKLAND

FIRST SEMESTER, 2008
Campus: Tamaki

COMPUTER SCIENCE

Data Communications Fundamentals

(Time allowed: 45 minutes)

NOTE:

- Attempt *all* questions in the space provided.
  Extra space for answers is available on page 5
- This mid-semester test will contribute 50% to your coursework mark,
  and 15% to your overall course mark.
  Indicated marks are out of a total of 100 marks (about two per minute).
- No marks will be awarded if you merely state a correct answer. To obtain full credit, your script must clearly explain *why* your answer is correct.
- *If you require additional information in order to answer a question, you should make a reasonable assumption as required for your answer, and you should explain your assumption on your script.*

Surname: ..... Forenames: .....

Student ID: .....

Question No.	*Possible marks*	*Awarded marks*
1. Coding	40	
2. Kraft's theorem	30	
3. Huffman's coding	30	
Total	100	

Student ID: .....

1. **Coding** [40 marks]

(a) Define the following notions:

i. Code [10 marks]

A code is the assignment of a unique string of characters (a codeword) to each character in an alphabet.

ii. Prefix Code [10 marks]

A prefix code (or prefix-free code) is a code in which no codeword is a proper prefix of another codeword.

iii. Uniquely Decodable Code [5 marks]

A code is uniquely decodable if the encoding of every possible cleartext using that code is unique.

(b) Is the assignment

$$1 \longrightarrow a$$
$$0 \longrightarrow b$$
$$10 \longrightarrow c$$
$$01 \longrightarrow d$$

a code? [10 marks]

The assignment is a code because different letters have been assigned different codewords.

(c) Is the assignment defined at (b) a prefix-free (prefix code)? [5 marks]

As 1 is a proper prefix of 10, the code in not prefix-free.

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2. **Kratf's theorem** [30 marks]

(a) State Kraft's theorem. [10 marks]

Kraft's theorem. A prefix code exists for codewords lengths $l_1, l_2, \ldots, l_N$ if and only if $2^{-l_1} + 2^{-l_2} + \cdots + 2^{-l_N} \leq 1$.

(b) Check whether the numbers 10, 2, 5 satisfy the inequality in Kraft's theorem. [10 marks]

$2^{-10} + 2^{-2} + 2^{-5} = 1/1024 + 1/4 + 1/32 < 1$, so the numbers 10, 2, 5 satisfy the inequality in Kraft's theorem.

(c) Construct a prefix binary code for which the codewords lengths are exactly: 10, 2, 5. [10 marks]

Reorder the lengths 10, 2, 5 in increasing order, 2,5,10 and then, using Kraft's construction, get the prefix code: $00, 01000, 1000000000$.

3. **Huffman's coding** [30 marks]

Devise two correct Huffman trees and their corresponding codewords for the letters A, B, C, D, E with frequencies given in the following table:

Letter	Frequency
A	15%
B	15%
C	10%
D	10%
E	50%

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Write the letters in increasing order of their frequencies, say C, D, A, B, E, and use Huffman's procedure:

C (10) D (10) A (15) B (15) E (50)
CD (20) AB (30) E (50)
CDAB (50) E (50)
CDABE (100)

so the code is

Letter	Code
A	101
B	100
C	111
D	110
E	0

If we write the letters in increasing order of their frequencies as D, C, A, B, E, and use Huffman's procedure:

D (10) C (10) A (15) B (15) E (50)
DC(20) AB (30) E (50)
DCAB (50) E (50)
DCABE (100)

so the code is

Letter	Code
A	101
B	100
C	110
D	111
E	0