THE UNIVERSITY OF AUCKLAND

EXAMINATION FOR BSc BScHons ETC 2003

COMPUTER SCIENCE: COMPSCI.220.S1T
Algorithms and Data Structures

(Time allowed: ONE hour)

Family Name (*please, print clearly*): ....

Given Name(s): ....

Degree (BSc, BSc(Hons), etc.): ....

Student Identification Number: ....

Signature: ....

Attempt *all* questions. Put the answers in the boxes below the questions. You may continue your answers onto the "overflow" pages provided at the back of the book if necessary.

Marks for each question are shown below and just before each answer box. Use of calculators is NOT permitted.

Section:	A	B	Total
Possible marks:	25	25	50
Awarded marks:			

QUESTION/ANSWER SHEETS FOLLOW

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1 Algorithm Analysis

1. Assume that each of the expressions below gives the processing time $T(n)$ spent by an algorithm for solving a problem of size n. Select the dominant term and specify the Big-Oh complexity of each algorithm. **[6 marks]**

   Hint: The dominant term has the steepest increase in n.

   ANSWER:

Expression	*Dominant term*	$O(\ldots)$
$500 + 100n + 25\log_{10} n$	$100n$	$O(n)$
$100 + 10n^{1.5} + n\log_{10} n$	$10n^{1.5}$	$O(n^{1.5}$
$3n + 5n^{3.5} + 30n^{2.5}$	$5n^{3.5}$	$O(n^{3.5}$
$10n\log_4 n + n(\log_2 n)^2 + 15n$	$n(\log_2 n)^2$	$O(n(\log n)^2)$
$2^n + n^{100} + n^n$	n^n	$O(n^n)$
$(\log_4 n)^2 + (\log_2 \log_2 n)^2$	$(\log_4 n)^2$	$O((\log n)^2)$

Expression	Dominant term	$O(\ldots)$
$500 + 100n + 25\log_{10} n$		
$100 + n^{1.5} + n\log_{10} n$		
$3n + 5n^{3.5} + 30n^{2.5}$		
$10n\log_4 n + n(\log_2 n)^2 + 15n$		
$2^n + n^{100} + n^n$		
$(\log_4 n)^2 + (\log_2 \log_2 n)^2$		

2. Prove that $f(n) = 60 + 30n + 3n^{1.5}$ is $O(n^{1.5})$ **[4 marks]**

   Hint: $f(n)$ is $O(g(n))$ if there exist a positive factor $c > 0$ and a

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positive threshold $n_0 > 0$ such that $f(n) \leq c \cdot g(n)$ for all $n \geq n_0$.

ANSWER:
$f(n) = 60 + 30n + 3n^{1.5} = n^{1.5}\left(\frac{60}{n^{1.5}} + \frac{30}{n^{0.5}} + 3\right)$, *or* $f(n) <$
$(60 + 30 + 3)n^{1.5} = 93n^{1.5}$ *if* $n > 1$

3. Solve the recurrence $T(n) = T(n/3) + 2n$; $T(0) = 0$, by "telescoping" **[6 marks]**

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Hint: assume $n = 3^m$. A useful formula: $1+3+\cdots+3^{k-1} = \frac{3^k-1}{3-1} \equiv \frac{1}{2}3^k$

ANSWER:

$$\begin{array}{lcl} T(3^m) & = & T(3^{m-1}) + 2\cdot 3^m \\ T(3^{m-1}) & = & T(3^{m-1}) + 2\cdot 3^{m-1} \\ \cdots & \cdots & \cdots \\ T(3^2) & = & T(3) + 2\cdot 3^2 \\ T(3) & = & T(1) + 2\cdot 3 \\ T(1) & = & T(0) + 2\cdot 1 \end{array}$$

or $T(3^m) = 2(1+3+3^2+\cdots+3^m) = 2\frac{3^{m+1}-1}{3-1}$, *or* $T(3^m) = 3^{m+1} - 1$. *Thus* $T(n) \approx 3n$.

4. The game "Find the number" is as follows. One player thinks of a number in the range from 1 to n. The other player has to find the number by asking questions of the form "is the number less than x?" The goal is to ask as few questions as possible, assuming that nobody cheats.

   (a) Design a good strategy for this game by specifying which values

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x should occur in each successive question. **[3 marks]**

ANSWER:
The desired strategy uses ideas of the binary search: Question 1: $x = n/2$, and depending on the answer, the like question to the left or right half of the range.

(b) Give and solve the basic recurrence for the number of questions

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$T(n)$ in the proposed strategy. **[3 marks]**

ANSWER:
$T(n) = T(n/2) + 1$; $T(1) = 0$, *and the solution by telescoping:*

$$\begin{array}{lcl} T(2^m) & = & T(2^{m-1}) + 1 \\ T(2^{m-1}) & = & T(2^{m-2}) + 1 \\ \cdots & \cdots & \cdots \\ T(2) & = & T(1) + 1 \end{array}$$

so that $T(2^m) = m$, *or* $T(n) = \log_2 n$

5. Compare the basic properties of a maximum heap and a binary search tree (BST) and find out which trees can be both a maximum heap and

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a BST. **[3 marks]**

ANSWER:
The heap is a complete binary tree, and the maximum heap property is that the key of each parent node is greater than or equal to the key of any child node. The BST property is that for every node x in the tree, the values of all the keys in the left subtree are smaller than or equal to the key in x, and the values of all the keys in the right subtree are greater than the key in x. Thus, the only maximum heaps that are also the BST are: the sole root, and the root with only the left child.

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2 Graph Algorithms

1. Answer the following questions for the digraph with the given adjacency lists representation.

0:	2		
1:	0		
2:	0	1	
3:	4	5	6
4:	5		
5:	3	4	6
6:	1	2	

   (a) What is the order?

   ANSWER:
   7

   (b) How many strongly connected components are there?

   ANSWER:
   3

   (c) What is the maximum indegree of a vertex?

   ANSWER:
   2

   (d) What is the distance from node 4 to node 2?

   ANSWER:
   3

CONTINUED

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(e) Is node 6 contained in a cycle?

ANSWER:
NO

[5 marks]

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2. Consider the digraph G with nodes $0, \ldots, 6$ whose adjacency matrix representation is given below.

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(a) Write down the adjacency lists representation of G.

[1 mark]

ANSWER:

0:	*1*	*4*	*5*
1:	*0*	*3*	
2:	*0*	*6*	
3:	*0*	*5*	
4:	*5*		
5:			
6:	*5*		

(b) Suppose that BFS is run on G, with the rule that whenever there is a choice of node to visit, the one with smallest label is chosen. List all tree arcs, forward arcs, back arcs, and cross arcs of G.

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[4 marks]

ANSWER:
Tree: $(0,1)(0,4)(0,5)(1,3)(2,6)$. *Forward: none. Back:* $(1,0)(3,0)$. *Cross:* $(2,0)(3,5)(4,5)(6,5)$.

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3. Suppose that DFS is run on a digraph G and the following timestamps obtained.

Node	0	1	2	3	4	5	6
Seen	0	8	11	4	2	9	1
Done	7	13	12	5	3	10	6

(a) How many trees are in the DFS forest?

ANSWER:
2

[3 marks]

(b) Is node 3 necessarily a descendant of node 6 in the DFS forest?

ANSWER:
YES

[1 mark]

(c) Assuming that G is a DAG, write down a topological order of the nodes of G.

ANSWER:
1 2 5 0 6 3 4

[2 marks]

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(d) Is it possible that G contains an arc $(3, 4)$?

ANSWER:
YES

[1 mark]

(e) Is it possible that G contains an arc $(5, 2)$?

ANSWER:
NO

[1 mark]

(f) Suppose that $(1, 6)$ is an arc of G. Which type of arc (tree, forward, back or cross) is it?

ANSWER:
cross

[2 marks]

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4. Answer each question TRUE or FALSE. Correct answers receive 1 mark; incorrect ones receive -0.5 marks.

   (a) If BFS is run on a graph, and there is a cross edge, then the graph has a cycle.

   ANSWER:
   TRUE

   (b) If a graph has a cycle, then when BFS is run, there will be a cross edge.

   ANSWER:
   TRUE

   (c) When DFS is run on a graph, every edge is a cross edge or a tree edge.

   ANSWER:
   FALSE

   (d) If DFS is run on a digraph and (v, w) is a cross arc, then v is seen before w.

   ANSWER:
   FALSE

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(e) If DFS is run on a digraph and v is visited before w, then w finishes processing before v.

ANSWER:
FALSE

[5 marks]

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Additional work pages

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Additional work pages