Lecture 21

21.1. Assume that $\mathbf{a} = (a_1, a_2)$ and $\mathbf{b} = (b_1, b_2)$ are two orthogonal vectors in $\mathbb{R}^2$. What is the inner product $\mathbf{a} \cdot \mathbf{b}$ in this case?

21.2. Assume that vector $\mathbf{t} = (t_1, t_2)$ is given and we are interested in a vector $\mathbf{n} = (n_1, n_2)$ which is orthogonal to $\mathbf{t}$ and of length 1 (i.e., a unit vector), and where $\mathbf{t}$ and $\mathbf{n}$ form a right-hand coordinate system. Is vector $\mathbf{n}$ uniquely specified by these conditions?

21.3. What are *tangent vector*, *unit normal vector*, and *radius of the osculating circle* of an arc or curve γ?

21.4. Define curvature $\kappa(t)$ of a curve γ by changes of the slope angle $\psi(t)$. What is the meaning of positive or negative values of curvature (defined this way)? How is the radius of the osculating circle at $\gamma(t)$ related to $\kappa(t)$?

21.5. What is the curvature of a circle of radius r, and of a straight segment?

21.6. What are two options for estimating curvature of a digital curve, following definitions of curvature for simple arcs or curves in $\mathbb{R}^2$?

21.7. How to define a *corner* based on curvature?

Lecture 22

22.1. Assume an 8-curve in $\mathbb{Z}^2$. What are *forward vector* and *backward vector* (defined by parameter k)?

22.2. How to estimate curvature based on directions of forward and backward vector at a pixel p on an 8-arc or 8-curve?

22.2. Apply the estimation (as defined in 22.2) manually to a short arc or curve, illustrating the principle by way of examples (say, on pixel of "low curvature", and one pixel of "high curvature").

22.3. Let $\theta_i = \theta_b/2 + \theta_f/2$, $\delta_f = |\theta_f - \theta_i|$, and $\delta_b = |\theta_b - \theta_i|$. Show that $\delta_f = \delta_b$, for two reals θ_b and θ_f.

22.4. Describe a method for estimating curvature based on maximum-length DSS calculations.

22.5. Consider an example for algorithm **M2003**, such as $x(0) = x_i = 8$, $x(1) = x_{i-3} = 5$, $x(2) = x_{i+3} = 10$, $y(0) = y_i = 10$, $y(1) = y_{i-3} = 11$, $y(2) = y_{i+3} = 8$. In this algorithm, we apply the second order polynomials

$$x(t) = a_0 + a_1 t + a_2 t^2$$
$$y(t) = b_0 + b_1 t + b_2 t^2$$

Solve this for a_0, a_1, a_2, b_0, b_1, and b_2, and calculate the curvature at pixel (x_i, y_i). Hint: use the formula

$$\kappa(t) = \frac{\dot{x}(t)\ddot{y}(t) \; - \; \dot{y}(t)\ddot{x}(t)}{\left(\dot{x}(t)^2 + \dot{y}(t)^2\right)^{-1 \cdot 5}}$$

Lecture 23

23.1. Explain the histogram of arc-chord distances, and illustrate it by means of examples: "a straight arc", "a convex corner", "a concave corner", and "a noisy arc".

23.2. Specify a corner detector based on analyzing histograms of arc-chord distances.

23.3. A significance measure (for corner detection) is defined in a region of support. What is the *region of support*? Provide one example of a significance measure.

23.4. Identify corners by analyzing all triangles p_{i-k_1}, p_i, p_{i+k_2} at p_i, with $k_1 + k_2 = n \geq 8$, and $k_1, k_2 \geq 3$. You may use the area and/or the angle at p_i (of a selected triangle from the set of all triangles at p_i) for identifying a corner.

23.5. Specify a corner detector based on calculating maximum-length DSSs.

Lecture 24

24.1. Define a set of curves for testing curvature estimators (by drawing a sketch of these curves). For each curve in your set provide arguments why this curve is of interest for this comparative performance test (Which particular feature can we obtain based on this curve?).

24.2. Assume we run a curvature estimator on a digital curve which gives us function of curvature values (defined for all pixels on the curve). Suggest a way to specify corners based on values of such a function.

24.3. Describe an experiment for analyzing multigrid convergence of estimated curvature values.

24.4. Suggest performance measures for evaluating corner detectors.

Lecture 25

25.1. Describe an algorithm for uniquely labeling all 6-components in a 3D picture (of size $l \times m \times n$).

25.2. What are *6-inner* and *6-border voxels* of a given 6-component of voxels.

25.3. What is the *frontier* of a set of voxels? As examples, consider (a) a single voxel, and (b) two 2-adjacent voxels.

25.4. Assume we have a single voxel in a 3D picture. How does this map into the 3D frontier grid?

25.5. What is a *Hamiltonian path*, a *Hamiltonian circuit*, and a *Hamiltonian graph*?

25.6. Is $[\mathbb{Z}^3, A_6]$ a Hamiltonian graph? Does it have a Hamiltonian path?

25.7. Illustrate by means of an example that the 2-cells of the frontier of a simple 2-arc of 3-cells do have a Hamiltonian path. Your example of an arc should have at least three 3-cells.

Lecture 26

26.1. Define the frontier graph of a 2-region of voxels (3-cells).

26.2. Give an example of a non-planar frontier graph of a 2-region.

26.3. Describe the FILL algorithm for visiting all nodes of a frontier graph of a 2-region. How often do we visit each node in the frontier graph when applying this algorithm?

26.4. Define the *out-faces* and *in-faces* for the set of all frontier faces of a 2-region. How many out-faces, and how many in-faces does one frontier face have?

26.5. How often do we visit each node in the frontier graph when applying the Artzy-Herman algorithm?

26.6. Would it be possible to design an algorithm for frontier tracing which visits each frontier face of a 2-region exactly once? Give reasons for your answer.

26.7. What are the asymptotic time complexities of the FILL algorithm and of the Artzy-Herman algorithm when tracing a frontier of n frontier faces?