Computer Science 773

Robotics and Real-time Control

PID CONTROLLERS

The recommended procedure for constructing a controller for a machine is to determine the behaviour of all the components of the system, decide on the desired behaviour of the machine, then work out what function of what input variables the controller must compute to give the deisred behaviour.

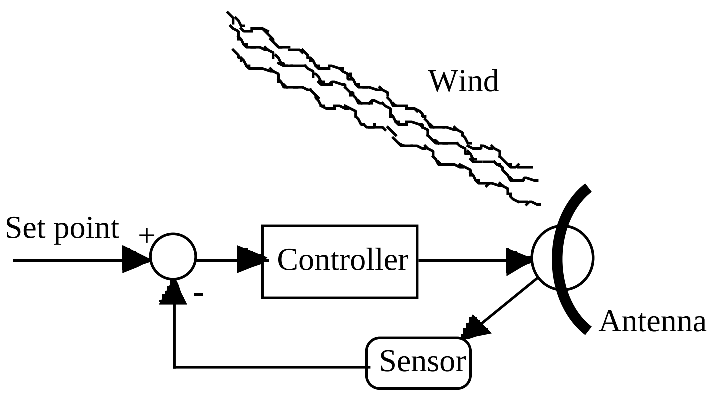
Unfortunately, as well as being recommended it is slow and expensive, and for simple and routine tasks like keeping a motor speed constant, or stabilising a dirigible antenna from vagaries of the wind and weather, or keeping the temperature and irrigation levels steady in a hydroponic plant, it would be much easier if we could just buy a box labelled "controller" and use it.

Fortunately, we can. It won't necessarily be as good as the purpose-built ideal controller, but in many circumstances it will do a good job. It's possible because in most circumstances a controller can be built wihout knowing the precise form of the machine's behaviour provided that the response to the control input is smooth and monotonic. In just the same way, you can make a reasonable job of driving a new car because you know qualitatively what the controls do, though you can do much better once you are more accustomed to it, and have learnt its response to the controls in much more detail.

In this sheet, I describe a common form of general-purpose controller, and try to explain why it contains the functions which make it work. I shall use the example of controlling a steerable radar antenna throughout; there's nothing particularly remarkable about the example, except that it's easy to visualise.

THE EXAMPLE, AND ITS CONTROL CIRCUIT.

To begin with, we describe the problem in a little more detail. We have a large radar dish-shaped antenna, which we wish to be able to point in any direction (so it can't be permanently mounted with rigid supports), but also wish to hold steady in a direction of interest for significant periods. We'll concentrate on the steady direction, in which we have a fixed set-point and wish to hold the direction steady. Redirecting the antenna is then not quite a simple matter of changing the set point (because the antenna can rotate for ever, we can't use a simple linear scale) but quite close to that.



The problem arises because the antenna will not just stand still. It is a large object, and the wind forces on it are considerable. A steady wind will tend to rotate the antenna from the desired direction, which a gusty wind will tend to jiggle it from side to side. Our controller must be able to cope with either of these contingencies.

STEP 1 : A PROPORTIONAL CONTROLLER.

The first, and obvious, step is to observe that if the antenna is pointing too far to the left, then the controller must push it towards the right. Of course, if the antenna is pointing in the right direction, then - other things being equal - the controller must leave it alone. The obvious controller function is therefore :

$$c = K e$$

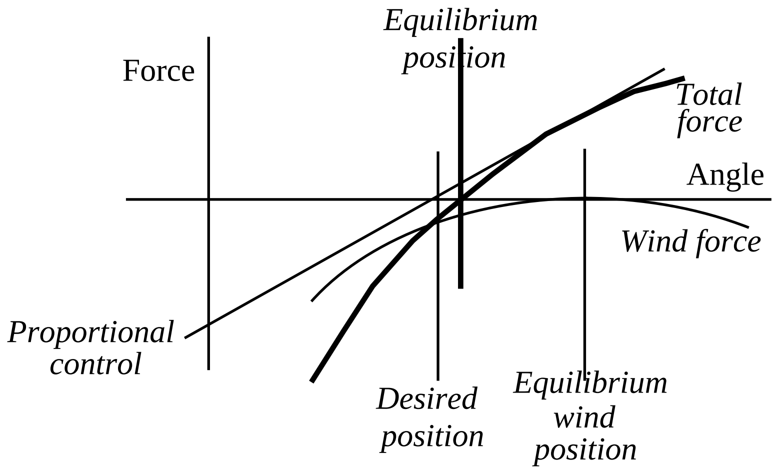
with the controller output, c, proportional to the error signal, e.

Now, we know that in fact things are not so simple (see *CONTROL THEORY PRINCIPLES*), but in practice friction will save us from perpetual oscillation, and a simple proportional controller isn't a bad start. That's roughly what the James Watt steam engine governor does. For directing radar antennae, though, proportional control isn't good enough.

STEP 2 : INTEGRATION.

Suppose now that our antenna is pointing in the required direction, but a steady wind is blowing. Unless we are exceedingly fortunate, the wind will impose a steady torque on the antenna which will tend to turn it from its correct orientation, so the controller must supply a counteracting force to keep the antenna still.

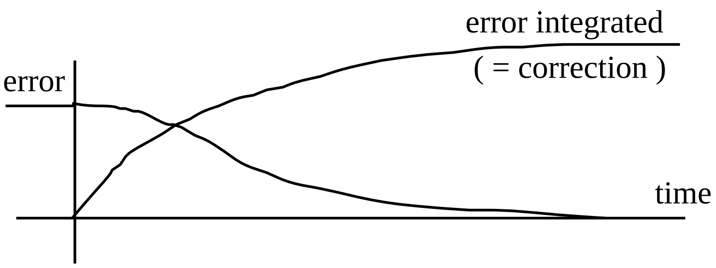
A proportional controller won't do that. What will happen ? Consider this graph.



There will be a position of equilibrium, but it won't be the position we want; clearly, proportional control alone isn't sufficient.

How can we tell that proportional control has failed ? It isn't simply that the antenna is pointing the wrong way; that's likely to happen under any circumstances as random events perturb the equilibrium. It is more important that the error is systematically either positive or negative. If we could detect this condition, we could use it as a signal to add something else to the proportional control output.

We can do that by integrating the error. If the integral of the error signal is added as a correction to the error signal itself (taking care with the sign !), the correction will increase until the error becomes zero, and, if the offset produced by the wind forces is constant, the corrected position will be stable. A change in wind direction will change the situation, and the integration will bring the antenna vack to its required position by establishing a new correction.



Notice that random fluctuations in the error signal will integrate to zero, so not affect the result. The new control function is now

$$c = K_1 e + K_2 \int e \, dt$$

$1 / K_2$ is called the *integration time*; it has the dimensions of time, and a magnitude comparable with the time it takes to approach the new steady state reasonably closely.

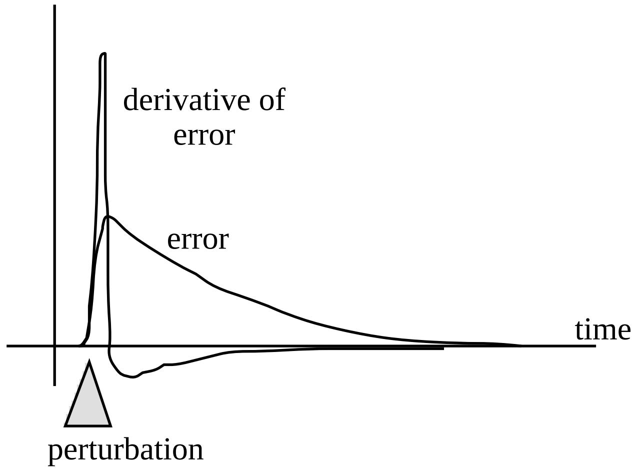
A controller in which the control signal is the sum of proportional and integral terms is called a PI controller. By choosing appropriate values for the constants (often called *gains*), it can be fitted to a wide variety of systems, and works reasonably well in most cases.

STEP 3 : DIFFERENTIATION.

But we have still not dealt with the gusty winds. Can we do anything to stabilise the antenna in the face of sudden perturbations resulting from sudden changes in wind velocity and direction ?

We can never achieve perfect control, because we have to wait until there is some perturbation before we know there's anything to correct. That makes it all the more important that we should be able to react quickly when a perturbation happens. What we'd like to do is to notice a sudden change when it happens, and react by applying a fairly large force for a short time in the opposite direction to counter the external disturbance.

To do so, we must be able to identify sudden changes quickly, and react accordingly. A sudden change in the error signal will appear as a steep rise or fall in the error value, which will therefore have a large (positive or negative) derivative. Adding a derivative component to the error signal might therefore be a way to achieve the desired behaviour.



The sharp increase in derivative can be fed back as a sharp impulsive force countering the original perturbation, so that the initial increase is stemmed and much reduced from the magnitude it would have in absence of the correction.

The control function for this behaviour is :

$$c = K_1\ e + K_2 \int e\ dt + K_3\ de/dt$$

K_3 is the *differentiation time*. The derivative function is rather more sensitive than the others, because it reacts very strongly to perturbations which are of negligible magnitude; without due care, tiny but sharp noise signals which might have very large derivatives can overwhelm impulses of much greater magnitude but slower rise times, which would nullify the value of the signal component. (The description in the textbook gives more discussion of these factors.)

The result is the PID controller (otherwise called the three-term controller), which is a very effective device for tourine control in a variety of environments.

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