# SUPPLEMENTARY MATERIAL FOR: IDENTIFICATION OF VECTOR AUTOREGRESSIVE MODELS WITH GRANGER AND STABILITY CONSTRAINTS

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## ABSTRACT

In this document, we present experimental results that have not been included in [1] due to space constraints. The notation is the same as in the main manuscript.

In the caption of each table within the supplementary material, we write down the name of the experiment in which the results have been obtained as well as the sample size ($N$) used in the training phase. In all cases, the total number of trials is $N_{\text{tr}} = 200$.

In Tables 1-8, we show statistics concerning how many times each order in the set $\{1, \ldots, p_{\max}\}$, where $p_{\max} = 15$, is selected by the IT criteria SBC, FPE, RNML and AICc. Note that in the first column of each table, the true VAR-order is written in bold.

In Figs. 1-2 (which are the same as Figs. 3-4 in [1]), we show boxplots for the average distances from the estimated sparsity pattern to the true one. These are graphical representations for the results in Tables 9-16. The convention in these tables is that the columns correspond to the IT criteria applied for VAR-order selection, while the rows correspond to the IT criteria used to choose the Granger sparsity pattern. For the results reported in the last column of such a table, we make the assumption that the order of the model is known. This is why the last column has the label "ORACLE".

Amongst the three estimation methods that we investigate, only "LS with Granger constraints" does not guarantee that the model fitted to the data is stable. In Tables 17-24, we give the empirical probability that models estimated with this method are not stable. The labels for the rows and for the columns are the same as in Tables 9-16. The only difference is that there is a row labeled "ORACLE", which corresponds to the case when the Granger sparsity pattern is assumed to be known.

For the description of the forecasting experiments, we refer to [1]. In this document, we reproduce Figs. 1-2 from [1], which are named Figs. 3-4. They are graphical representations of the results reported in Tables 25-32. Note that the significance of the IT criteria assigned to the rows and to the columns of the Tables 25-32 is the same as in the Tables 17-24.

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# VAR-order estimation

| VAR Order | IT Criterion | | | |
|---|---|---|---|---|
| | SBC | FPE | RNML | AICc |
| 1 | 44 | 0 | 44 | 0 |
| 2 | 56 | 0 | 63 | 4 |
| **3** | 100 | 200 | 93 | 196 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 | 0 |

**Table 1**: $\text{Exp}_1$: $N = 200$

| VAR Order | IT Criterion | | | |
|---|---|---|---|---|
| | SBC | FPE | RNML | AICc |
| 1 | 9 | 0 | 8 | 0 |
| 2 | 34 | 0 | 26 | 0 |
| **3** | 157 | 200 | 166 | 200 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 | 0 |

**Table 2**: $\text{Exp}_1$: $N = 300$

| VAR Order | IT Criterion | | | |
|---|---|---|---|---|
| | SBC | FPE | RNML | AICc |
| 1 | 3 | 0 | 3 | 0 |
| 2 | 9 | 0 | 5 | 0 |
| **3** | 188 | 200 | 192 | 200 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 | 0 |

**Table 3**: $\text{Exp}_1$: $N = 400$

| VAR Order | IT Criterion | | | |
|---|---|---|---|---|
| | SBC | FPE | RNML | AICc |
| 1 | 0 | 0 | 1 | 0 |
| 2 | 5 | 0 | 1 | 0 |
| **3** | 195 | 200 | 198 | 200 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 | 0 |

**Table 4**: $\text{Exp}_1$: $N = 500$

| VAR Order | IT Criterion | | | |
|---|---|---|---|---|
| | SBC | FPE | RNML | AICc |
| 1 | 7 | 0 | 3 | 0 |
| 2 | 9 | 0 | 10 | 0 |
| 3 | 20 | 0 | 20 | 0 |
| 4 | 15 | 0 | 11 | 0 |
| 5 | 29 | 0 | 22 | 4 |
| 6 | 19 | 1 | 19 | 1 |
| 7 | 22 | 4 | 23 | 14 |
| 8 | 24 | 12 | 24 | 20 |
| 9 | 24 | 26 | 26 | 39 |
| **10** | 31 | 147 | 42 | 122 |
| 11 | 0 | 7 | 0 | 0 |
| 12 | 0 | 2 | 0 | 0 |
| 13 | 0 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 | 0 |
| 15 | 0 | 1 | 0 | 0 |

**Table 5**: $\text{Exp}_2$: $N = 200$

| VAR Order | IT Criterion | | | |
|---|---|---|---|---|
| | SBC | FPE | RNML | AICc |
| 1 | 2 | 0 | 1 | 0 |
| 2 | 4 | 0 | 2 | 0 |
| 3 | 8 | 0 | 5 | 0 |
| 4 | 7 | 0 | 6 | 0 |
| 5 | 14 | 0 | 15 | 0 |
| 6 | 17 | 0 | 9 | 0 |
| 7 | 22 | 1 | 15 | 3 |
| 8 | 23 | 4 | 21 | 10 |
| 9 | 38 | 19 | 38 | 26 |
| **10** | 65 | 169 | 88 | 161 |
| 11 | 0 | 5 | 0 | 0 |
| 12 | 0 | 1 | 0 | 0 |
| 13 | 0 | 1 | 0 | 0 |
| 14 | 0 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 | 0 |

**Table 6**: $\text{Exp}_2$: $N = 300$

| VAR Order | IT Criterion | | | |
|---|---|---|---|---|
| | SBC | FPE | RNML | AICc |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 1 | 0 | 0 | 0 |
| 3 | 3 | 0 | 1 | 0 |
| 4 | 5 | 0 | 2 | 0 |
| 5 | 10 | 0 | 8 | 0 |
| 6 | 9 | 0 | 6 | 0 |
| 7 | 16 | 0 | 9 | 0 |
| 8 | 18 | 3 | 21 | 4 |
| 9 | 39 | 9 | 32 | 15 |
| **10** | 99 | 183 | 121 | 180 |
| 11 | 0 | 3 | 0 | 1 |
| 12 | 0 | 2 | 0 | 0 |
| 13 | 0 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 | 0 |

**Table 7**: $\text{Exp}_2$: $N = 400$

| VAR Order | IT Criterion | | | |
|---|---|---|---|---|
| | SBC | FPE | RNML | AICc |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 2 | 0 | 1 | 0 |
| 4 | 1 | 0 | 1 | 0 |
| 5 | 5 | 0 | 6 | 0 |
| 6 | 4 | 0 | 4 | 0 |
| 7 | 17 | 0 | 8 | 0 |
| 8 | 17 | 1 | 17 | 3 |
| 9 | 29 | 9 | 26 | 8 |
| **10** | 125 | 178 | 137 | 185 |
| 11 | 0 | 8 | 0 | 4 |
| 12 | 0 | 2 | 0 | 0 |
| 13 | 0 | 1 | 0 | 0 |
| 14 | 0 | 1 | 0 | 0 |
| 15 | 0 | 0 | 0 | 0 |

**Table 8**: $\text{Exp}_2$: $N = 500$

## Average distance from the estimated sparsity pattern $\widehat{\mathcal{G}}$ to the true sparsity pattern $\mathcal{G}$

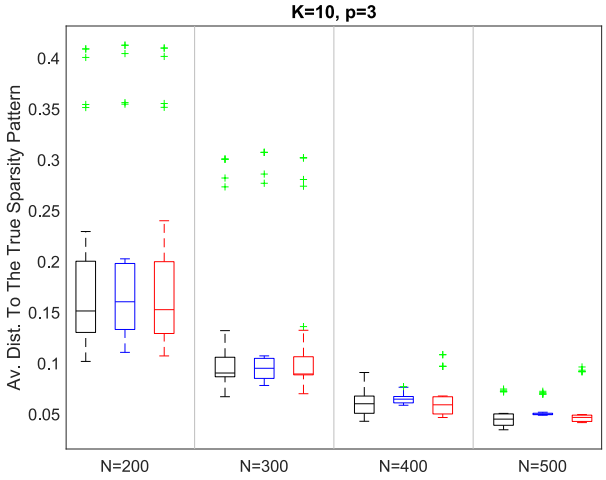


**Fig. 1**: $\text{Exp}_1$: Average distances from the estimated sparsity pattern to the true one. For each sample size ($N$), a different color is used for each boxplot, depending on the convex optimization problem solved in the training phase: the first boxplot (in black) is for LS with Granger constraints, the second one (in blue) is for the estimation method from [2] and the last one (in red) is for the iterative estimation method.

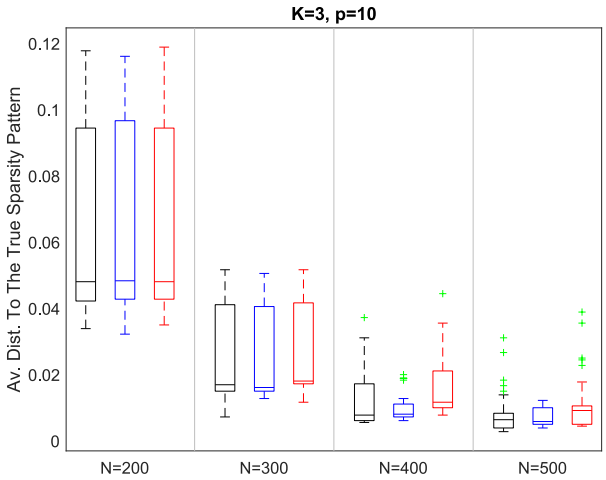


**Fig. 2**: $\text{Exp}_2$: Average distances from the estimated sparsity pattern to the true one. All graphical conventions are the same as in Fig. 1.

| LS with Granger constraints | | | | | |
|---|---|---|---|---|---|
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.156 | 0.102 | 0.174 | 0.103 | 0.102 |
| FPE | 0.215 | 0.130 | 0.229 | 0.131 | 0.130 |
| RNML | 0.158 | 0.105 | 0.171 | 0.105 | 0.105 |
| AICc | 0.351 | 0.409 | 0.354 | 0.400 | 0.409 |
| EBIC | 0.187 | 0.145 | 0.198 | 0.145 | 0.145 |
| ERNML | 0.189 | 0.146 | 0.200 | 0.146 | 0.146 |
| Estimation method from [2] | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.176 | 0.133 | 0.190 | 0.133 | 0.133 |
| FPE | 0.173 | 0.111 | 0.198 | 0.112 | 0.111 |
| RNML | 0.175 | 0.132 | 0.188 | 0.132 | 0.132 |
| AICc | 0.355 | 0.412 | 0.356 | 0.404 | 0.412 |
| EBIC | 0.191 | 0.146 | 0.203 | 0.146 | 0.146 |
| ERNML | 0.192 | 0.147 | 0.202 | 0.147 | 0.147 |
| Iterative estimation method | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.161 | 0.107 | 0.179 | 0.108 | 0.107 |
| FPE | 0.215 | 0.129 | 0.240 | 0.130 | 0.129 |
| RNML | 0.162 | 0.108 | 0.175 | 0.108 | 0.108 |
| AICc | 0.351 | 0.410 | 0.355 | 0.401 | 0.410 |
| EBIC | 0.188 | 0.143 | 0.198 | 0.143 | 0.143 |
| ERNML | 0.189 | 0.144 | 0.200 | 0.144 | 0.144 |

**Table 9**: $\text{Exp}_1$: $N = 200$

| LS with Granger constraints | | | | | |
|---|---|---|---|---|---|
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.088 | 0.067 | 0.088 | 0.067 | 0.067 |
| FPE | 0.132 | 0.103 | 0.127 | 0.103 | 0.103 |
| RNML | 0.087 | 0.067 | 0.085 | 0.067 | 0.067 |
| AICc | 0.273 | 0.300 | 0.282 | 0.300 | 0.300 |
| EBIC | 0.105 | 0.089 | 0.103 | 0.089 | 0.089 |
| ERNML | 0.106 | 0.090 | 0.103 | 0.090 | 0.090 |
| Estimation method from [2] | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.102 | 0.085 | 0.102 | 0.085 | 0.085 |
| FPE | 0.101 | 0.078 | 0.101 | 0.078 | 0.078 |
| RNML | 0.100 | 0.084 | 0.099 | 0.084 | 0.084 |
| AICc | 0.277 | 0.307 | 0.286 | 0.307 | 0.307 |
| EBIC | 0.106 | 0.090 | 0.104 | 0.090 | 0.090 |
| ERNML | 0.107 | 0.091 | 0.105 | 0.091 | 0.091 |
| Iterative estimation method | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.089 | 0.070 | 0.088 | 0.070 | 0.070 |
| FPE | 0.132 | 0.106 | 0.136 | 0.106 | 0.106 |
| RNML | 0.089 | 0.070 | 0.087 | 0.070 | 0.070 |
| AICc | 0.274 | 0.302 | 0.281 | 0.302 | 0.302 |
| EBIC | 0.105 | 0.088 | 0.102 | 0.088 | 0.088 |
| ERNML | 0.106 | 0.089 | 0.102 | 0.089 | 0.089 |

**Table 10**: $\text{Exp}_1$: $N = 300$

| LS with Granger constraints | | | | | |
|---|---|---|---|---|---|
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.056 | 0.050 | 0.056 | 0.050 | 0.050 |
| FPE | 0.091 | 0.082 | 0.090 | 0.082 | 0.082 |
| RNML | 0.056 | 0.051 | 0.056 | 0.051 | 0.051 |
| AICc | 0.053 | 0.043 | 0.052 | 0.043 | 0.043 |
| EBIC | 0.069 | 0.064 | 0.068 | 0.064 | 0.064 |
| ERNML | 0.068 | 0.064 | 0.067 | 0.064 | 0.064 |
| Estimation method from [2] | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.066 | 0.061 | 0.066 | 0.061 | 0.061 |
| FPE | 0.076 | 0.067 | 0.077 | 0.067 | 0.067 |
| RNML | 0.065 | 0.060 | 0.065 | 0.060 | 0.060 |
| AICc | 0.067 | 0.059 | 0.067 | 0.059 | 0.059 |
| EBIC | 0.069 | 0.064 | 0.068 | 0.064 | 0.064 |
| ERNML | 0.069 | 0.064 | 0.068 | 0.064 | 0.064 |
| Iterative estimation method | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.055 | 0.050 | 0.055 | 0.050 | 0.050 |
| FPE | 0.108 | 0.097 | 0.108 | 0.097 | 0.097 |
| RNML | 0.056 | 0.050 | 0.055 | 0.050 | 0.050 |
| AICc | 0.055 | 0.047 | 0.053 | 0.047 | 0.047 |
| EBIC | 0.067 | 0.062 | 0.067 | 0.062 | 0.062 |
| ERNML | 0.068 | 0.063 | 0.067 | 0.063 | 0.063 |

**Table 11**: $\text{Exp}_1$: $N = 400$

| LS with Granger constraints | | | | | |
|---|---|---|---|---|---|
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.040 | 0.040 | 0.041 | 0.040 | 0.040 |
| FPE | 0.073 | 0.072 | 0.074 | 0.072 | 0.072 |
| RNML | 0.040 | 0.039 | 0.040 | 0.039 | 0.039 |
| AICc | 0.036 | 0.034 | 0.037 | 0.034 | 0.034 |
| EBIC | 0.050 | 0.049 | 0.050 | 0.049 | 0.049 |
| ERNML | 0.051 | 0.049 | 0.050 | 0.049 | 0.049 |
| Estimation method from [2] | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.051 | 0.050 | 0.051 | 0.050 | 0.050 |
| FPE | 0.071 | 0.070 | 0.072 | 0.070 | 0.070 |
| RNML | 0.050 | 0.049 | 0.050 | 0.049 | 0.049 |
| AICc | 0.051 | 0.049 | 0.052 | 0.049 | 0.049 |
| EBIC | 0.050 | 0.049 | 0.050 | 0.049 | 0.049 |
| ERNML | 0.051 | 0.049 | 0.050 | 0.049 | 0.049 |
| Iterative estimation method | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.045 | 0.044 | 0.045 | 0.044 | 0.044 |
| FPE | 0.093 | 0.092 | 0.096 | 0.092 | 0.092 |
| RNML | 0.042 | 0.042 | 0.043 | 0.042 | 0.042 |
| AICc | 0.044 | 0.042 | 0.045 | 0.042 | 0.042 |
| EBIC | 0.049 | 0.048 | 0.049 | 0.048 | 0.048 |
| ERNML | 0.049 | 0.048 | 0.049 | 0.048 | 0.048 |

**Table 12**: $\text{Exp}_1$: $N = 500$

| LS with Granger constraints | | | | | |
|---|---|---|---|---|---|
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.098 | 0.043 | 0.091 | 0.049 | 0.044 |
| FPE | 0.109 | 0.037 | 0.098 | 0.038 | 0.034 |
| RNML | 0.094 | 0.041 | 0.091 | 0.047 | 0.042 |
| AICc | 0.081 | 0.034 | 0.073 | 0.036 | 0.038 |
| EBIC | 0.118 | 0.044 | 0.102 | 0.050 | 0.044 |
| ERNML | 0.117 | 0.044 | 0.102 | 0.051 | 0.045 |
| Estimation method from [2] | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.107 | 0.043 | 0.097 | 0.048 | 0.043 |
| FPE | 0.084 | 0.032 | 0.078 | 0.033 | 0.035 |
| RNML | 0.103 | 0.041 | 0.099 | 0.048 | 0.042 |
| AICc | 0.088 | 0.039 | 0.083 | 0.043 | 0.041 |
| EBIC | 0.116 | 0.044 | 0.102 | 0.051 | 0.045 |
| ERNML | 0.116 | 0.044 | 0.103 | 0.051 | 0.045 |
| Iterative estimation method | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.101 | 0.044 | 0.097 | 0.050 | 0.045 |
| FPE | 0.108 | 0.036 | 0.094 | 0.036 | 0.035 |
| RNML | 0.094 | 0.042 | 0.094 | 0.046 | 0.043 |
| AICc | 0.083 | 0.037 | 0.078 | 0.038 | 0.042 |
| EBIC | 0.119 | 0.046 | 0.105 | 0.051 | 0.046 |
| ERNML | 0.119 | 0.046 | 0.104 | 0.052 | 0.046 |

**Table 13**: $\text{Exp}_2$: $N = 200$

| LS with Granger constraints | | | | | |
|---|---|---|---|---|---|
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.046 | 0.014 | 0.038 | 0.016 | 0.015 |
| FPE | 0.052 | 0.019 | 0.043 | 0.019 | 0.018 |
| RNML | 0.046 | 0.014 | 0.038 | 0.015 | 0.015 |
| AICc | 0.032 | 0.008 | 0.027 | 0.007 | 0.009 |
| EBIC | 0.051 | 0.015 | 0.041 | 0.016 | 0.016 |
| ERNML | 0.051 | 0.015 | 0.042 | 0.016 | 0.016 |
| Estimation method from [2] | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.047 | 0.016 | 0.041 | 0.017 | 0.016 |
| FPE | 0.034 | 0.013 | 0.029 | 0.015 | 0.013 |
| RNML | 0.047 | 0.015 | 0.041 | 0.016 | 0.016 |
| AICc | 0.033 | 0.014 | 0.029 | 0.015 | 0.014 |
| EBIC | 0.051 | 0.015 | 0.041 | 0.016 | 0.016 |
| ERNML | 0.051 | 0.016 | 0.041 | 0.017 | 0.016 |
| Iterative estimation method | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.047 | 0.017 | 0.039 | 0.018 | 0.017 |
| FPE | 0.050 | 0.021 | 0.044 | 0.020 | 0.018 |
| RNML | 0.047 | 0.017 | 0.040 | 0.017 | 0.017 |
| AICc | 0.035 | 0.013 | 0.031 | 0.012 | 0.013 |
| EBIC | 0.052 | 0.017 | 0.042 | 0.017 | 0.017 |
| ERNML | 0.050 | 0.017 | 0.042 | 0.017 | 0.017 |

**Table 14**: $\text{Exp}_2$: $N = 300$

| LS with Granger constraints | | | | | |
|---|---|---|---|---|---|
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.017 | 0.006 | 0.009 | 0.006 | 0.006 |
| FPE | 0.037 | 0.017 | 0.031 | 0.017 | 0.017 |
| RNML | 0.016 | 0.006 | 0.009 | 0.006 | 0.006 |
| AICc | 0.021 | 0.006 | 0.011 | 0.006 | 0.006 |
| EBIC | 0.019 | 0.006 | 0.010 | 0.006 | 0.006 |
| ERNML | 0.019 | 0.006 | 0.010 | 0.006 | 0.006 |
| Estimation method from [2] | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.018 | 0.008 | 0.011 | 0.008 | 0.008 |
| FPE | 0.020 | 0.009 | 0.013 | 0.009 | 0.008 |
| RNML | 0.019 | 0.007 | 0.011 | 0.007 | 0.007 |
| AICc | 0.019 | 0.008 | 0.011 | 0.008 | 0.007 |
| EBIC | 0.019 | 0.006 | 0.010 | 0.006 | 0.006 |
| ERNML | 0.019 | 0.006 | 0.010 | 0.006 | 0.006 |
| Iterative estimation method | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.022 | 0.010 | 0.014 | 0.012 | 0.010 |
| FPE | 0.044 | 0.021 | 0.036 | 0.023 | 0.021 |
| RNML | 0.022 | 0.010 | 0.014 | 0.012 | 0.010 |
| AICc | 0.027 | 0.009 | 0.017 | 0.011 | 0.009 |
| EBIC | 0.022 | 0.010 | 0.013 | 0.010 | 0.010 |
| ERNML | 0.020 | 0.008 | 0.011 | 0.008 | 0.008 |

**Table 15**: $\text{Exp}_2$: $N = 400$

| LS with Granger constraints | | | | | |
|---|---|---|---|---|---|
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.007 | 0.005 | 0.008 | 0.004 | 0.003 |
| FPE | 0.031 | 0.018 | 0.027 | 0.017 | 0.015 |
| RNML | 0.008 | 0.004 | 0.008 | 0.003 | 0.003 |
| AICc | 0.014 | 0.006 | 0.012 | 0.006 | 0.006 |
| EBIC | 0.008 | 0.005 | 0.008 | 0.004 | 0.003 |
| ERNML | 0.008 | 0.005 | 0.008 | 0.004 | 0.003 |
| Estimation method from [2] | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.010 | 0.007 | 0.010 | 0.006 | 0.005 |
| FPE | 0.012 | 0.006 | 0.011 | 0.004 | 0.004 |
| RNML | 0.011 | 0.007 | 0.011 | 0.006 | 0.005 |
| AICc | 0.010 | 0.006 | 0.010 | 0.004 | 0.004 |
| EBIC | 0.009 | 0.006 | 0.009 | 0.005 | 0.004 |
| ERNML | 0.009 | 0.006 | 0.009 | 0.004 | 0.004 |
| Iterative estimation method | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.009 | 0.006 | 0.010 | 0.005 | 0.004 |
| FPE | 0.039 | 0.025 | 0.036 | 0.024 | 0.023 |
| RNML | 0.011 | 0.006 | 0.011 | 0.005 | 0.004 |
| AICc | 0.018 | 0.009 | 0.015 | 0.008 | 0.008 |
| EBIC | 0.011 | 0.006 | 0.011 | 0.005 | 0.004 |
| ERNML | 0.010 | 0.006 | 0.010 | 0.005 | 0.004 |

**Table 16**: $\text{Exp}_2$: $N = 500$

# Empirical probability that the estimated model is not stable

| LS with Granger constraints | | | | | |
|---|---|---|---|---|---|
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.105 | 0.170 | 0.095 | 0.175 | 0.170 |
| FPE | 0.110 | 0.165 | 0.100 | 0.165 | 0.165 |
| RNML | 0.100 | 0.165 | 0.095 | 0.170 | 0.165 |
| AICc | 0.080 | 0.140 | 0.075 | 0.140 | 0.140 |
| EBIC | 0.135 | 0.175 | 0.115 | 0.180 | 0.175 |
| ERNML | 0.150 | 0.180 | 0.140 | 0.185 | 0.180 |
| ORACLE | 0.100 | 0.160 | 0.095 | 0.160 | 0.160 |

**Table 17**: $\text{Exp}_1$: $N = 200$

| LS with Granger constraints | | | | | |
|---|---|---|---|---|---|
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.115 | 0.135 | 0.115 | 0.135 | 0.135 |
| FPE | 0.085 | 0.110 | 0.095 | 0.110 | 0.110 |
| RNML | 0.095 | 0.115 | 0.105 | 0.115 | 0.115 |
| AICc | 0.095 | 0.115 | 0.100 | 0.115 | 0.115 |
| EBIC | 0.100 | 0.125 | 0.105 | 0.125 | 0.125 |
| ERNML | 0.095 | 0.125 | 0.105 | 0.125 | 0.125 |
| ORACLE | 0.075 | 0.105 | 0.085 | 0.105 | 0.105 |

**Table 18**: $\text{Exp}_1$: $N = 300$

| LS with Granger constraints | | | | | |
|---|---|---|---|---|---|
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.110 | 0.110 | 0.110 | 0.110 | 0.110 |
| FPE | 0.090 | 0.095 | 0.090 | 0.095 | 0.095 |
| RNML | 0.115 | 0.110 | 0.110 | 0.110 | 0.110 |
| AICc | 0.095 | 0.100 | 0.095 | 0.100 | 0.100 |
| EBIC | 0.115 | 0.115 | 0.115 | 0.115 | 0.115 |
| ERNML | 0.115 | 0.115 | 0.115 | 0.115 | 0.115 |
| ORACLE | 0.080 | 0.085 | 0.075 | 0.085 | 0.085 |

**Table 19**: $\text{Exp}_1$: $N = 400$

| LS with Granger constraints | | | | | |
|---|---|---|---|---|---|
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.085 | 0.085 | 0.080 | 0.085 | 0.085 |
| FPE | 0.075 | 0.075 | 0.070 | 0.075 | 0.075 |
| RNML | 0.095 | 0.095 | 0.085 | 0.095 | 0.095 |
| AICc | 0.080 | 0.080 | 0.075 | 0.080 | 0.080 |
| EBIC | 0.100 | 0.100 | 0.090 | 0.100 | 0.100 |
| ERNML | 0.100 | 0.100 | 0.090 | 0.100 | 0.100 |
| ORACLE | 0.070 | 0.070 | 0.065 | 0.070 | 0.070 |

**Table 20**: $\text{Exp}_1$: $N = 500$

| LS with Granger constraints | | | | | |
|---|---|---|---|---|---|
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.140 | 0.305 | 0.150 | 0.305 | 0.325 |
| FPE | 0.155 | 0.300 | 0.170 | 0.300 | 0.325 |
| RNML | 0.140 | 0.305 | 0.150 | 0.305 | 0.325 |
| AICc | 0.140 | 0.305 | 0.150 | 0.305 | 0.325 |
| EBIC | 0.140 | 0.305 | 0.150 | 0.305 | 0.325 |
| ERNML | 0.140 | 0.305 | 0.145 | 0.305 | 0.325 |
| ORACLE | 0.140 | 0.295 | 0.150 | 0.295 | 0.315 |

**Table 21**: $\text{Exp}_2$: $N = 200$

| LS with Granger constraints | | | | | |
|---|---|---|---|---|---|
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.195 | 0.255 | 0.205 | 0.255 | 0.265 |
| FPE | 0.195 | 0.255 | 0.205 | 0.260 | 0.270 |
| RNML | 0.195 | 0.255 | 0.205 | 0.255 | 0.265 |
| AICc | 0.195 | 0.255 | 0.205 | 0.255 | 0.265 |
| EBIC | 0.195 | 0.255 | 0.205 | 0.255 | 0.265 |
| ERNML | 0.195 | 0.255 | 0.205 | 0.255 | 0.265 |
| ORACLE | 0.195 | 0.255 | 0.205 | 0.255 | 0.265 |

**Table 22**: $\text{Exp}_2$: $N = 300$

| LS with Granger constraints | | | | | |
|---|---|---|---|---|---|
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.180 | 0.245 | 0.210 | 0.250 | 0.250 |
| FPE | 0.185 | 0.245 | 0.205 | 0.250 | 0.250 |
| RNML | 0.180 | 0.245 | 0.210 | 0.250 | 0.250 |
| AICc | 0.185 | 0.245 | 0.210 | 0.250 | 0.250 |
| EBIC | 0.180 | 0.245 | 0.210 | 0.250 | 0.250 |
| ERNML | 0.180 | 0.245 | 0.210 | 0.250 | 0.250 |
| ORACLE | 0.180 | 0.245 | 0.210 | 0.250 | 0.250 |

**Table 23**: $\text{Exp}_2$: $N = 400$

| LS with Granger constraints | | | | | |
|---|---|---|---|---|---|
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 0.130 | 0.150 | 0.125 | 0.150 | 0.150 |
| FPE | 0.125 | 0.150 | 0.120 | 0.150 | 0.150 |
| RNML | 0.130 | 0.150 | 0.125 | 0.150 | 0.150 |
| AICc | 0.125 | 0.150 | 0.120 | 0.150 | 0.150 |
| EBIC | 0.130 | 0.150 | 0.125 | 0.150 | 0.150 |
| ERNML | 0.130 | 0.150 | 0.125 | 0.150 | 0.150 |
| ORACLE | 0.130 | 0.150 | 0.125 | 0.150 | 0.150 |

**Table 24**: $\text{Exp}_2$: $N = 500$

# Prediction errors

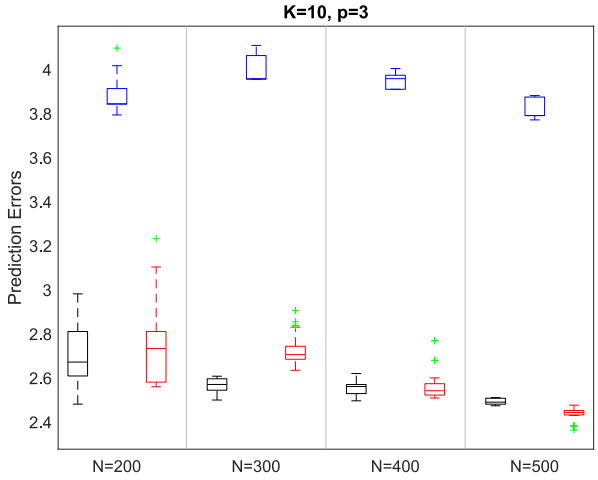


**Fig. 3**: Results of prediction in $\text{Exp}_1$. This plot is the same as the one in [1, Fig. 1].

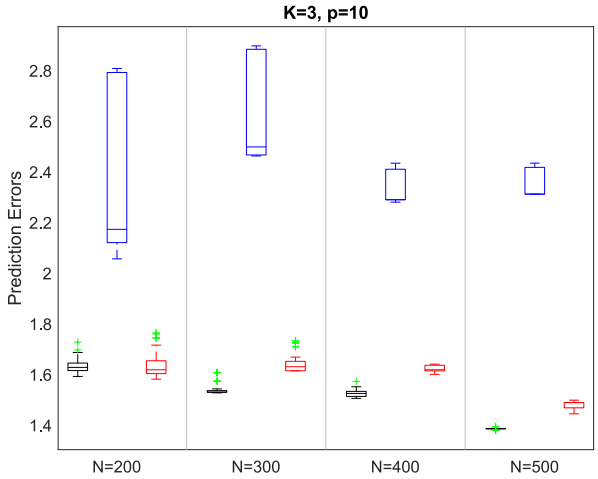


**Fig. 4**: Results of prediction in $\text{Exp}_2$. This plot is the same as the one in [1, Fig. 2].

| LS with Granger constraints | | | | | |
|---|---|---|---|---|---|
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 2.668 | 2.549 | 2.690 | 2.549 | 2.549 |
| FPE | 2.711 | 2.666 | 2.757 | 2.666 | 2.666 |
| RNML | 2.672 | 2.597 | 2.694 | 2.597 | 2.597 |
| AICc | 2.675 | 2.642 | 2.656 | 2.642 | 2.642 |
| EBIC | 2.793 | 2.816 | 2.858 | 2.816 | 2.816 |
| ERNML | 2.888 | 2.866 | 2.981 | 2.866 | 2.866 |
| ORACLE | 2.694 | 2.481 | 2.695 | 2.481 | 2.481 |
| Estimation method from [2] | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 3.994 | 3.842 | 3.914 | 3.842 | 3.842 |
| FPE | 3.993 | 3.853 | 3.914 | 3.845 | 3.853 |
| RNML | 3.994 | 3.842 | 3.909 | 3.842 | 3.842 |
| AICc | 3.886 | 3.793 | 3.886 | 3.793 | 3.793 |
| EBIC | 4.004 | 3.842 | 3.908 | 3.842 | 3.842 |
| ERNML | 4.004 | 3.842 | 3.908 | 3.842 | 3.842 |
| ORACLE | 4.097 | 3.844 | 4.017 | 3.844 | 3.844 |
| Iterative estimation method | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 2.723 | 2.560 | 2.877 | 2.560 | 2.560 |
| FPE | 2.733 | 2.564 | 2.815 | 2.564 | 2.564 |
| RNML | 2.728 | 2.597 | 2.948 | 2.597 | 2.597 |
| AICc | 2.798 | 2.624 | 2.938 | 2.624 | 2.624 |
| EBIC | 2.835 | 2.751 | 3.103 | 2.751 | 2.751 |
| ERNML | 2.860 | 2.766 | 3.232 | 2.766 | 2.766 |
| ORACLE | 2.785 | 2.575 | 3.101 | 2.575 | 2.575 |

**Table 25**: $\mathrm{Exp}_1$: $N = 200$

| LS with Granger constraints | | | | | |
|---|---|---|---|---|---|
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 2.513 | 2.558 | 2.558 | 2.558 | 2.558 |
| FPE | 2.501 | 2.607 | 2.501 | 2.607 | 2.607 |
| RNML | 2.542 | 2.596 | 2.590 | 2.596 | 2.596 |
| AICc | 2.550 | 2.600 | 2.602 | 2.600 | 2.600 |
| EBIC | 2.550 | 2.570 | 2.550 | 2.570 | 2.570 |
| ERNML | 2.570 | 2.590 | 2.570 | 2.590 | 2.590 |
| ORACLE | 2.499 | 2.541 | 2.517 | 2.541 | 2.541 |
| Estimation method from [2] | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 4.063 | 3.955 | 4.087 | 3.955 | 3.955 |
| FPE | 4.063 | 3.957 | 4.087 | 3.957 | 3.957 |
| RNML | 4.063 | 3.955 | 4.087 | 3.955 | 3.955 |
| AICc | 4.047 | 3.958 | 4.060 | 3.958 | 3.958 |
| EBIC | 4.063 | 3.955 | 4.109 | 3.955 | 3.955 |
| ERNML | 4.063 | 3.955 | 4.109 | 3.955 | 3.955 |
| ORACLE | 4.060 | 3.958 | 4.083 | 3.958 | 3.958 |
| Iterative estimation method | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 2.767 | 2.742 | 2.837 | 2.742 | 2.742 |
| FPE | 2.702 | 2.690 | 2.745 | 2.690 | 2.690 |
| RNML | 2.718 | 2.698 | 2.839 | 2.698 | 2.698 |
| AICc | 2.713 | 2.682 | 2.778 | 2.682 | 2.682 |
| EBIC | 2.733 | 2.662 | 2.830 | 2.662 | 2.662 |
| ERNML | 2.733 | 2.634 | 2.854 | 2.634 | 2.634 |
| ORACLE | 2.744 | 2.706 | 2.906 | 2.706 | 2.706 |

**Table 26**: $\mathrm{Exp}_1$: $N = 300$

| LS with Granger constraints | | | | | |
|---|---|---|---|---|---|
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 2.614 | 2.570 | 2.594 | 2.570 | 2.570 |
| FPE | 2.565 | 2.554 | 2.565 | 2.554 | 2.554 |
| RNML | 2.620 | 2.561 | 2.594 | 2.561 | 2.561 |
| AICc | 2.496 | 2.496 | 2.496 | 2.496 | 2.496 |
| EBIC | 2.582 | 2.558 | 2.570 | 2.558 | 2.558 |
| ERNML | 2.602 | 2.570 | 2.582 | 2.570 | 2.570 |
| ORACLE | 2.559 | 2.505 | 2.520 | 2.505 | 2.505 |
| Estimation method from [2] | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 3.973 | 3.910 | 3.942 | 3.910 | 3.910 |
| FPE | 3.989 | 3.958 | 3.989 | 3.958 | 3.958 |
| RNML | 3.973 | 3.910 | 3.942 | 3.910 | 3.910 |
| AICc | 3.989 | 3.958 | 3.989 | 3.958 | 3.958 |
| EBIC | 4.004 | 3.910 | 3.968 | 3.910 | 3.910 |
| ERNML | 4.004 | 3.910 | 3.968 | 3.910 | 3.910 |
| ORACLE | 4.003 | 3.958 | 3.988 | 3.958 | 3.958 |
| Iterative estimation method | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 2.588 | 2.522 | 2.559 | 2.522 | 2.522 |
| FPE | 2.579 | 2.508 | 2.544 | 2.508 | 2.508 |
| RNML | 2.588 | 2.534 | 2.563 | 2.534 | 2.534 |
| AICc | 2.563 | 2.522 | 2.540 | 2.522 | 2.522 |
| EBIC | 2.769 | 2.542 | 2.680 | 2.542 | 2.542 |
| ERNML | 2.768 | 2.542 | 2.680 | 2.542 | 2.542 |
| ORACLE | 2.600 | 2.522 | 2.577 | 2.522 | 2.522 |

**Table 27**: $\mathrm{Exp}_1$: $N = 400$

| LS with Granger constraints | | | | | |
|---|---|---|---|---|---|
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 2.508 | 2.508 | 2.476 | 2.508 | 2.508 |
| FPE | 2.481 | 2.481 | 2.478 | 2.481 | 2.481 |
| RNML | 2.476 | 2.508 | 2.501 | 2.508 | 2.508 |
| AICc | 2.510 | 2.510 | 2.479 | 2.510 | 2.510 |
| EBIC | 2.473 | 2.490 | 2.490 | 2.490 | 2.490 |
| ERNML | 2.473 | 2.490 | 2.490 | 2.490 | 2.490 |
| ORACLE | 2.493 | 2.493 | 2.493 | 2.493 | 2.493 |
| Estimation method from [2] | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 3.874 | 3.790 | 3.874 | 3.790 | 3.790 |
| FPE | 3.874 | 3.771 | 3.874 | 3.771 | 3.771 |
| RNML | 3.874 | 3.790 | 3.874 | 3.790 | 3.790 |
| AICc | 3.874 | 3.790 | 3.874 | 3.790 | 3.790 |
| EBIC | 3.881 | 3.790 | 3.881 | 3.790 | 3.790 |
| ERNML | 3.881 | 3.790 | 3.881 | 3.790 | 3.790 |
| ORACLE | 3.871 | 3.788 | 3.871 | 3.788 | 3.788 |
| Iterative estimation method | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 2.433 | 2.443 | 2.452 | 2.443 | 2.443 |
| FPE | 2.432 | 2.432 | 2.432 | 2.432 | 2.432 |
| RNML | 2.433 | 2.443 | 2.452 | 2.443 | 2.443 |
| AICc | 2.452 | 2.443 | 2.452 | 2.443 | 2.443 |
| EBIC | 2.429 | 2.462 | 2.476 | 2.462 | 2.462 |
| ERNML | 2.429 | 2.462 | 2.476 | 2.462 | 2.462 |
| ORACLE | 2.364 | 2.382 | 2.382 | 2.382 | 2.382 |

**Table 28**: $\mathrm{Exp}_1$: $N = 500$

| LS with Granger constraints | | | | | |
|---|---|---|---|---|---|
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 1.675 | 1.605 | 1.624 | 1.635 | 1.616 |
| FPE | 1.728 | 1.601 | 1.628 | 1.611 | 1.612 |
| RNML | 1.667 | 1.615 | 1.634 | 1.645 | 1.616 |
| AICc | 1.696 | 1.606 | 1.634 | 1.635 | 1.616 |
| EBIC | 1.687 | 1.615 | 1.641 | 1.645 | 1.616 |
| ERNML | 1.687 | 1.615 | 1.641 | 1.645 | 1.616 |
| ORACLE | 1.681 | 1.592 | 1.631 | 1.611 | 1.612 |
| Estimation method from [2] | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 2.792 | 2.120 | 2.792 | 2.173 | 2.056 |
| FPE | 2.792 | 2.120 | 2.800 | 2.145 | 2.056 |
| RNML | 2.792 | 2.120 | 2.792 | 2.173 | 2.056 |
| AICc | 2.802 | 2.120 | 2.808 | 2.173 | 2.056 |
| EBIC | 2.792 | 2.120 | 2.792 | 2.173 | 2.056 |
| ERNML | 2.792 | 2.120 | 2.792 | 2.173 | 2.056 |
| ORACLE | 2.794 | 2.119 | 2.793 | 2.143 | 2.056 |
| Iterative estimation method | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 1.743 | 1.598 | 1.623 | 1.618 | 1.616 |
| FPE | 1.745 | 1.594 | 1.632 | 1.598 | 1.581 |
| RNML | 1.715 | 1.603 | 1.642 | 1.628 | 1.616 |
| AICc | 1.745 | 1.598 | 1.624 | 1.605 | 1.610 |
| EBIC | 1.763 | 1.603 | 1.650 | 1.628 | 1.616 |
| ERNML | 1.763 | 1.603 | 1.655 | 1.628 | 1.616 |
| ORACLE | 1.759 | 1.592 | 1.658 | 1.601 | 1.599 |

**Table 29**: $\mathrm{Exp}_2$: $N = 200$

| LS with Granger constraints | | | | | |
|---|---|---|---|---|---|
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 1.574 | 1.530 | 1.536 | 1.530 | 1.527 |
| FPE | 1.606 | 1.535 | 1.543 | 1.535 | 1.535 |
| RNML | 1.607 | 1.530 | 1.536 | 1.530 | 1.527 |
| AICc | 1.607 | 1.530 | 1.536 | 1.530 | 1.527 |
| EBIC | 1.574 | 1.530 | 1.536 | 1.530 | 1.527 |
| ERNML | 1.574 | 1.530 | 1.536 | 1.530 | 1.527 |
| ORACLE | 1.607 | 1.530 | 1.536 | 1.530 | 1.527 |
| Estimation method from [2] | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 2.897 | 2.466 | 2.883 | 2.498 | 2.462 |
| FPE | 2.897 | 2.466 | 2.883 | 2.497 | 2.462 |
| RNML | 2.897 | 2.466 | 2.883 | 2.498 | 2.462 |
| AICc | 2.897 | 2.466 | 2.883 | 2.497 | 2.462 |
| EBIC | 2.897 | 2.466 | 2.883 | 2.498 | 2.462 |
| ERNML | 2.897 | 2.466 | 2.883 | 2.498 | 2.462 |
| ORACLE | 2.897 | 2.466 | 2.883 | 2.497 | 2.462 |
| Iterative estimation method | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 1.724 | 1.614 | 1.652 | 1.614 | 1.627 |
| FPE | 1.709 | 1.618 | 1.631 | 1.614 | 1.630 |
| RNML | 1.732 | 1.614 | 1.652 | 1.614 | 1.627 |
| AICc | 1.709 | 1.614 | 1.631 | 1.614 | 1.627 |
| EBIC | 1.724 | 1.614 | 1.652 | 1.614 | 1.627 |
| ERNML | 1.724 | 1.614 | 1.652 | 1.614 | 1.627 |
| ORACLE | 1.732 | 1.635 | 1.668 | 1.635 | 1.652 |

**Table 30**: $\text{Exp}_2$: $N = 300$

| LS with Granger constraints | | | | | |
|---|---|---|---|---|---|
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 1.551 | 1.520 | 1.533 | 1.505 | 1.525 |
| FPE | 1.572 | 1.520 | 1.537 | 1.505 | 1.525 |
| RNML | 1.551 | 1.520 | 1.533 | 1.505 | 1.525 |
| AICc | 1.551 | 1.520 | 1.533 | 1.505 | 1.525 |
| EBIC | 1.551 | 1.520 | 1.533 | 1.505 | 1.525 |
| ERNML | 1.551 | 1.520 | 1.533 | 1.505 | 1.525 |
| ORACLE | 1.551 | 1.510 | 1.533 | 1.505 | 1.511 |
| Estimation method from [2] | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 2.431 | 2.290 | 2.410 | 2.290 | 2.280 |
| FPE | 2.431 | 2.290 | 2.410 | 2.290 | 2.280 |
| RNML | 2.430 | 2.290 | 2.410 | 2.290 | 2.280 |
| AICc | 2.431 | 2.290 | 2.410 | 2.290 | 2.280 |
| EBIC | 2.434 | 2.290 | 2.410 | 2.290 | 2.280 |
| ERNML | 2.434 | 2.290 | 2.410 | 2.290 | 2.280 |
| ORACLE | 2.431 | 2.290 | 2.410 | 2.290 | 2.280 |
| Iterative estimation method | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 1.615 | 1.614 | 1.604 | 1.631 | 1.600 |
| FPE | 1.638 | 1.618 | 1.619 | 1.631 | 1.605 |
| RNML | 1.615 | 1.631 | 1.604 | 1.631 | 1.617 |
| AICc | 1.615 | 1.614 | 1.604 | 1.631 | 1.600 |
| EBIC | 1.615 | 1.631 | 1.604 | 1.631 | 1.617 |
| ERNML | 1.641 | 1.641 | 1.619 | 1.641 | 1.640 |
| ORACLE | 1.641 | 1.641 | 1.616 | 1.641 | 1.640 |

**Table 31**: $\text{Exp}_2$: $N = 400$

| LS with Granger constraints | | | | | |
|---|---|---|---|---|---|
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 1.386 | 1.386 | 1.386 | 1.386 | 1.394 |
| FPE | 1.379 | 1.386 | 1.379 | 1.379 | 1.380 |
| RNML | 1.386 | 1.386 | 1.386 | 1.386 | 1.394 |
| AICc | 1.379 | 1.386 | 1.379 | 1.379 | 1.380 |
| EBIC | 1.386 | 1.386 | 1.386 | 1.386 | 1.394 |
| ERNML | 1.386 | 1.386 | 1.386 | 1.386 | 1.394 |
| ORACLE | 1.386 | 1.386 | 1.386 | 1.386 | 1.394 |
| Estimation method from [2] | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 2.434 | 2.312 | 2.417 | 2.312 | 2.312 |
| FPE | 2.434 | 2.312 | 2.417 | 2.312 | 2.312 |
| RNML | 2.434 | 2.312 | 2.417 | 2.312 | 2.312 |
| AICc | 2.434 | 2.312 | 2.417 | 2.312 | 2.312 |
| EBIC | 2.434 | 2.312 | 2.417 | 2.312 | 2.312 |
| ERNML | 2.434 | 2.312 | 2.417 | 2.312 | 2.312 |
| ORACLE | 2.434 | 2.312 | 2.417 | 2.312 | 2.312 |
| Iterative estimation method | | | | | |
| | SBC | FPE | RNML | AICc | ORACLE |
| SBC | 1.478 | 1.498 | 1.447 | 1.489 | 1.489 |
| FPE | 1.451 | 1.498 | 1.444 | 1.479 | 1.479 |
| RNML | 1.478 | 1.498 | 1.447 | 1.489 | 1.489 |
| AICc | 1.465 | 1.498 | 1.447 | 1.489 | 1.489 |
| EBIC | 1.478 | 1.498 | 1.447 | 1.489 | 1.489 |
| ERNML | 1.478 | 1.498 | 1.447 | 1.489 | 1.489 |
| ORACLE | 1.478 | 1.498 | 1.447 | 1.489 | 1.489 |

**Table 32**: $\mathrm{Exp}_2$: $N = 500$

## 1. REFERENCES


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[2] N. Raksasri and J. Songsiri, "Guaranteed stability of autoregressive models with Granger causality learned from Wald tests," *Engineering Journal*, vol. 21, no. 6, pp. 23–36, 2017.