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Computational Complementarity for Probabilistic Automata

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Abstract

Motivated by Mermin's analysis of *Einstein-Podolsky-Rosen* correlations [25] and [6] we study two computational complementarity principles introduced in [7] for a class of probabilistic automata. We prove the existence of probabilistic automata featuring both types of computational complementarity and we present a method to reduce, under certain conditions, the study of computational complementarity of probabilistic automata to the study of computational complementarity of deterministic automata.

1 Introduction

Quantum entanglement [33] and nonlocal correlations [1, 16] are essential features of quantized systems used for quantum computation [13]. Building on Moore's "Gedanken" experiments, in [34, 31] complementarity was modeled by means of finite automata. Two new type of computational complementarity principles have been introduced and studied in [7, 11, 12, 10, 37, 36] using Moore automata *without initial states*. Motivated by Mermin's simple device [25] designed to explain *Einstein-Podolsky-Rosen (EPR)* correlations and the analysis in [6], we study the above mentioned computational complementarity properties for a class of probabilistic automata. We prove the existence of probabilistic automata featuring both types of computational complementarity and we reduce the study of computational complementarity of probabilistic automata to the study of computational complementarity of deterministic automata.

2 Mermin's Device

Mermin [25] imagined a simple device to illustrate the EPR conundrum without using the classical quantum mechanical notions of wave functions, superposition, wave-particle duality, uncertainty principle, etc. The device has three "completely unconnected"[1] parts, two detectors (D1) and (D2) and a source (S) emitting particles. The source is

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[1]There are no relevant causal connections, neither mechanical nor electromagnetic or any other.

placed between the detectors: whenever a button is pushed on (S), shortly thereafter two particles emerge, moving off toward detectors (D1) and (D2). Each detector has a switch that can be set in one of three possible positions–labeled 1,2,3–and a bulb that can flash a red (R) or a green (G) light. The purpose of lights is to "communicate" information to the observer. Each detector flashes either red or green whenever a particle reaches it. Because of the lack of any relevant connections between any parts of the device, the link between the emission of particles by (S), i.e., as a result of pressing a button, and the subsequent flashing of detectors can only be provided by the passage of particles from (S) to (D1) and (D2). Additional tools can be used to check and confirm the lack of any communication, cf. [25], p. 941.

The device is repeatedly operated as follows:

1. the switch of either detector (D1) and (D2) is set randomly to 1 or 2 or 3, i.e., the settings or states 11, 12, 13, 21, 22, 23, 31, 32, 33 are equally likely,
2. pushing a button on (S) determines the emission toward both (D1) and (D2),
3. sometime later, (D1) and (D2) flash one of their lights, G or R,
4. every run is recorded in the form $ijXY$, meaning that D1 was set to state i and flashed X and (D2) was set to j and flashed Y.

For example, the record $31GR$ means "(D1) was set to 3 and flashed G and (D2) was set to 1 and flashed R".

Long recorded runs show the following pattern:

a) In records starting with ii, i.e., 11, 22, 33, both (D1) and (D2) flash the same colours, RR, GG, with equal frequency; RG and GR are never flashed.

b) In records starting with $ij, i \neq j$, i.e., 12, 13, 21, 23, 31, 32, both (D1) and (D2) flash the same colour only 1/4 of the time (RR and GG come with equal frequencies); the other 3/4 of the time, they flash different colours (RG, GR), occurring again with equal frequencies.

The above patterns are statistical, that is they are subject to usual fluctuations expected in every statistical prediction: patterns are more and more "visible" as the number of runs becomes larger and larger.

The conundrum posed by the existence of Mermin's device reveals as soon as we notice that the seemingly simplest physical explanation of the pattern a) is incompatible with pattern b). Indeed, as (D1) and (D2) are unconnected there is no way for one detector to "know", at any time, the state of the other detector or which colour the other is flashing. Consequently, it seems plausible to assume that the colour flashed by detectors is determined only by some property, or group of properties, of particles, say speed, size, shape, etc. What properties determine the colour does not really matter; only the fact that each particle carries a "program" which determines which colour a detector will flash in some state is important. So, we are led to the following two hypotheses:

H1 *Particles are classified into eight categories:*

$$GGG, GGR, GRG, GRR, RGG, RGR, RRG, RRR.$$[2]

H2 *Two particles produced in a given run carry identical programs.*

According to H1–H2, if particles produced in a run are of type RGR, then both detectors will flash R in states 1 and 3; they will flash G if both are in state 2. Detectors flash the same colours when being in the same states because *particles carry the same programs.*

It is clear that from H1–H2 it follows that *programs carried by particles do not depend in any way on the specific states of detectors*: they are properties of particles not of detectors. Consequently, both particles carry the same program whether or not detectors (D1) and (D2) are in the same states.[3]

One can easily argue that

[L] For each type of particle, *in runs of type* b) *both detectors flash the same colour at least one third of the time.*

The conundrum reveals as a significant difference appears between the data dictated by particle programs (colours agree at least one third of the time) and the quantum mechanical prediction (colours agree only one fourth of the time):

under H1–H2, *the observed pattern b) is incompatible with* [L].

3 Mermin's Probabilistic Automata

Consider now a probabilistic automaton simulating Mermin's device. The states of the automaton are all combinations of states of detectors (D1) and (D2), $Q = \{11, 12, 13, 21, 22, 23, 31, 32, 33\}$, the input alphabet models the lights red and green, $\Sigma = \{G, R\}$, the output alphabet captures all combinations of lights flashed by (D1) and (D2), $O = \{GG, GR, RG, RR\}$, and the output function $f : Q \to O$, modeling all combinations of green/red lights flashed by (D1) and (D2) in all their possible states, is probabilistically defined by:

$$\begin{aligned}
f(ii) &= XX, \text{ with probability } 1/2, \text{ for } i = 1,2,3, X \in \{G, R\},\\
f(ii) &= XY, \text{ with probability } 0, \text{ for } i = 1,2,3, X, Y \in \{G, R\}, X \neq Y,\\
f(ij) &= XX, \text{ with probability } 1/8, \text{ for } i, j = 1,2,3, i \neq j, X \in \{G, R\},\\
f(ij) &= XY, \text{ with probability } 3/8, \text{ for } i, j = 1,2,3, i \neq j, X, Y \in \{G, R\}, X \neq Y.
\end{aligned}$$

[2] A particle of type XYZ will cause a detector in state 1 to flash X; a detector in state 2 will flash Y and a detector in state 3 will flash Z.

[3] The emitting source (S) has no knowledge about the states of (D1) and (D2) and there is no communication among any parts of the device.

For example, $f(11) = RR$ with probability 1/2, $f(11) = GR$ with probability 0, $f(11) = RG$ with probability 0, $f(11) = RR$ with probability 1/2, $f(12) = GG$ with probability 1/8, $f(12) = GR$ with probability 3/8, $f(12) = RG$ with probability 3/8, $f(12) = RR$ with probability 1/8, etc.

The automaton transition $\delta : Q \times \Sigma \to Q$ is *not specified.* In fact, varying all transition functions δ we get a class of Mermin automata:

$$\mathcal{M} = (Q, \Sigma, O, \delta, (p_{ij}^{XY}, i, j = 1, 2, 3, X, Y \in \{G, R\})),$$

where p_{ij}^{XY} is the probability that the automaton on state ij outputs XY: $p_{ii}^{XX} = 1/2$, $p_{ii}^{XY} = 0$, $X \neq Y$, $p_{ij}^{XX} = 1/8$, $p_{ij}^{XY} = 3/8$, $X \neq Y$.

4 Computational Complementarity for Deterministic Automata

Moore [26] has studied some experiments on finite deterministic automata[4] trying to understand what kind of conclusions about the internal conditions of a finite machine could possibly be drawn from input-output experiments. A (simple) Moore experiment can be described as follows: a copy of the machine will be experimentally observed, i.e., the experimenter will input a finite sequence of input symbols to the machine and will observe the sequence of output symbols. The correspondence between input and output symbols depends on the particular chosen machine and on its initial state. The experimenter will study the sequences of input and output symbols and will try to conclude that "the machine being experimented on was in state q at the beginning of the experiment".

A state p is "indistinguishable" from a state q (with respect to a given automaton) if every experiment performed on the automaton starting in state p produces the same outcome as it would starting in state q.

Moore [26] has proven the following important result: *There exists an automaton such that any pair of its distinct states are distinguishable, but there is no experiment which can determine what state the machine was in at the beginning of the experiment.* In Calude, Calude, Svozil and Yu [7] two non-equivalent concepts of computational complementarity were introduced and studied for finite automata. Informally, they can be expressed as follows. Consider the class of all elements of reality (observables) and the following properties:

A Any two distinct elements of reality can be mutually distinguished by a suitably chosen measurement procedure, see Bridgman [4].

B For any element of reality, there exists a measurement which distinguishes between this element and all the others. That is, a distinction between any one of them and all the others is operational.

C There exists a measurement which distinguishes between any two elements of reality. That is, a single pre-defined experiment operationally exists to distinguish between an arbitrary pair of elements of reality.

Clearly, **C** implies **B** and **B** implies **A**, but both converse implications fail to be true; consequently, two *principles of complementarity* emerge:

[4] See Brauer [3], Hopcroft and Ullman [22] for good introductions into automata theory.

CI Property **A** *but not property* **B** *(and therefore not* **C***):* The elements of reality can be mutually distinguished by experiments, but one of these elements cannot be distinguished from all the other ones by any single experiment.

CII Property **B** *but not property* **C***:* Any element of reality can be distinguished from all the other ones by a single experiment, but there does not exist a single experiment which distinguishes between any pair of distinct elements.

We may regard *CI* as an "uncertainty principle" (cf. Conway [14, p. 21]), later termed "computational complementarity" by Finkelstein and Finkelstein [18].[5]

In *CII* each experiment "generates" a pair of distinct states which exercise a mutual influence, namely, they cannot be separated into proper independent parts by the experiment; this influence mimics, in a sense, the state of *quantum entanglement*.[6] We may be conceive *CII* as a *toy model* for the *EPR effect* (see [17, 28, 29]), as well as for the *Zou-Wang-Mandel effect* [40, 38, 19]. Under *CII*, for each experiment w we have at least two states q, q' (as distant as we like in terms of the emitting outputs) which interact via the experiment w: any measurement of q is affecting q' and, conversely, any measurement of q' is affecting q.

5 Computational Complementarity for Probabilistic Automata

Motivated by Mermin's automaton probabilistic automaton analysis in [6] we introduce and study computational complementarity for a class of probabilistic automata. In opposition with a more popular model, in which the transition is stochastic, but the output is deterministic (see [27]), here we will work with automata having deterministic transitions but stochastic outputs. So, our probabilistic finite automata consist of a finite set of input symbols (the alphabet), a finite set of states, a finite set of outputs, a set of transitions from state to state that occur on input symbols chosen from the alphabet and an *output probabilistic function*. For each symbol there is exactly one transition out of each state, possible back to the state itself. The output function emits an output with some probability.

Formally, a *finite probabilistic automaton* $\mathcal{A} = (Q, \Sigma, O, \delta, (a_{p,o})_{p \in Q, o \in O})$ consists of an input alphabet Σ, a finite set Q of states, an output finite set O, a transition function $\delta : Q \times \Sigma \to Q$ and an output probabilistic function $f : Q \to O$ given by the matrix $(a_{p,o})_{p \in Q, o \in O}$ satisfying the condition $\sum_{o \in O} a_{p,o} = 1$, for every $p \in Q$. The output emitted by $\mathcal{A}$ on p is $f(p) = o$ with probability $a_{p,o}$. In case of a deterministic finite automaton, for every $p \in Q$, there exist one (unique) output $o \in O$ such that $a_{p,o} = 1$ and all other probabilities are 0; that is, $f(p) = o$. As in case of deterministic automata the transition function extends naturally to $Q \times \Sigma^* \to Q$ satisfying the equation $\delta(p, uv) = \delta(\delta(p, u), v)$, for all $p \in Q, u, v \in \Sigma^*$.[7]

[5] These type of models have been intensively studied from the point of view of their experimental logical structure by Grib and Zapatrin [20, 21], Svozil [34], Schaller and Svozil [30, 31, 32], Dvurečenskij, Pulmannová and Svozil [15], Calude and Lipponen [8], Calude, Calude and Ştefănescu [9], Jurvanen, and Lipponen [24]. See Svozil and Zapatrin [35] for a comparison of models.

[6] In particular, this influence cannot be used to send an actual message from a state to the other.

[7] In what follows the extension will also be denoted by δ.

Consider, for example, the automaton consisting of $Q = \{1,2,3,4\}$, $\Sigma = \{0,1\}$, $O = \{G,R\}$, the transition given by the following tables

q	σ	$\delta(q,\sigma)$
1	0	4
1	1	3
2	0	1
2	1	3

q	σ	$\delta(q,\sigma)$
3	0	4
3	1	4
4	0	2
4	1	2

and the output function defined by

$$\begin{array}{ll} f(1) = G, \text{ with probability } 2/3, & f(3) = G, \text{ with probability } 1/3, \\ f(1) = R, \text{ with probability } 1/3, & f(3) = R, \text{ with probability } 2/3, \\ f(2) = G, \text{ with probability } 1/3, & f(4) = G, \text{ with probability } 1/3, \\ f(2) = R, \text{ with probability } 2/3, & f(4) = R, \text{ with probability } 2/3. \end{array}$$

The following graphical representation will be consistently used in what follows:

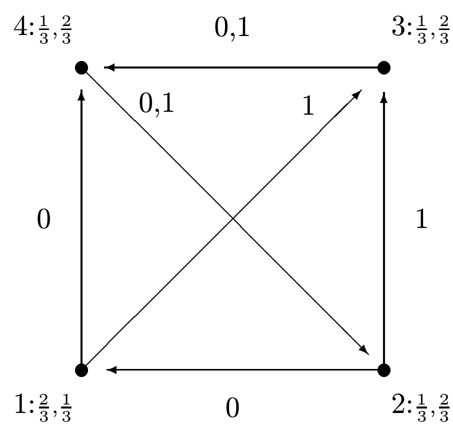


Figure 1.

The *response* of the automaton on state p to the "experiment" $x = x_1x_2\cdots x_n \in \Sigma^*$ is defined by a concatenation of random variables:

$$Res_{\mathcal{A}}(p, x_1x_2\cdots x_n) = f(p)f(\delta(p,x_1))\cdots f(\delta(p,x_1x_2\cdots x_n)).$$

For example, considering the automaton in Figure 1, the experiment starting in state 1 with input sequence 000100010 leads to the response:

$$Res_{\mathcal{A}}(1, 000100010) = f(1)f(4)f(2)f(1)f(3)f(4)f(2)f(1)f(3)f(4).$$

The response is $Res_{\mathcal{A}}(1, 000100010) = GRGGGRGGGR$ with probability $\cdot\frac{2}{3} = 2^6 \cdot 3^{-10}$; $Res_{\mathcal{A}}(1, 000100010) = GGGGGGGGGG$ with probability $8 \cdot 3^{-10}$, a.s.o.

Let $\alpha \in [1/2, 1]$. We say that a state p is *distinguishable* from the state q with *confidence* α if there exists an experiment $x = x_1 x_2 \cdots x_n \in \Sigma^*$ such that at least one probability that $f(p) \neq f(q)$, $f(\delta(p, x_1 x_2 \cdots x_i)) \neq f(\delta(q, x_1 x_2 \cdots x_i)), 1leqileqn$ is greater or equal to α.

This means that at least on one point, during the "measurement" process of the responses of the automaton to the experiment x, the probability that the response of $\mathcal{A}$ on p and x is different (within the fixed level of confidence) to the response of $\mathcal{A}$ on q and x.[8]

For the automaton in Figure 1 we have:

$$Res_{\mathcal{A}}(1, 001) = f(1)f(4)f(2)f(3), \qquad Res_{\mathcal{A}}(3, 001) = f(3)f(4)f(2)f(3),$$
$$Res_{\mathcal{A}}(2, 001) = f(2)f(1)f(4)f(2), \qquad Res_{\mathcal{A}}(4, 001) = f(4)f(2)f(1)f(3),$$

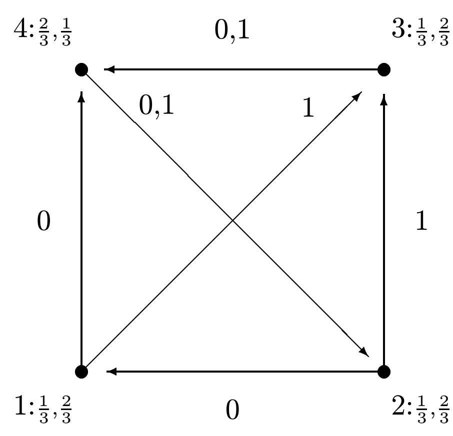
and the probability that $f(1) \neq f(i)$ is 5/9 for $i = 2, 3, 4$. So, with confidence 5/9 the experiment 001 distinguishes between every pair of distinct states.

We are now in a position to define properties **A, B, C** for a probabilistic automaton and a level of confidence α ($\alpha \in [1/2, 1]$).

- A probabilistic automaton has property **A** with level of confidence α if every pair of different states is distinguishable with *confidence* α.
- A probabilistic automaton has **B** with level of confidence α if every state is distinguishable with *confidence* α from any other distinct state.
- A probabilistic automaton has **C** with level of confidence α if there exists an experiment distinguishing with *confidence* α between each different states.

For example, the automaton in Figure 1 has **C**. Here are two examples of probabilistic automata having respectively **A** but not **B** and **B** but not **C**, i.e, *CI*, *CII*.

For *CI* we keep all components of the automaton in Figure 1 but modify the output function (see Figure 2):



[8]The motivation is the following. For a deterministic automaton $\mathcal{B}$, two states p, q are distinguishable if $Res_{\mathcal{B}}(p, x_1 \ldots x_n) \neq Res_{\mathcal{B}}(q, x_1 \ldots x_n)$, for some experiment $x_1 \ldots x_n$ (see [7]), that is, for the corresponding transition and output functions, $f(p) \neq f(q)$ or $f(\delta(p, x_1 x_2 \cdots x_i)) \neq f(\delta(q, x_1 x_2 \cdots x_i)), 1leqileqn$. In the probabilistic case we just replace the above conditions with the corresponding probabilistic ones.

Figure 2.

With confidence 5/9 the probabilistic automaton in Figure 2 has *CI*. Indeed, using the experiments 1, 10 and 010 we can distinguish with confidence 5/9 between every two distinct states (0 distinguishes between (1,2), (1,4), (2,3), 1 distinguishes between (1,3), 10 distinguishes between (2,4) and 010 distinguishes between (3,4)), but no experiment starting with 1 distinguishes with confidence 5/9 between states 1 and 2, and no experiment starting with 0 distinguishes between the states 1 and 3 ($\delta(1, 1x) = \delta(2, 1x) = \delta(3, x)$, $\delta(1, 0y) = \delta(3, 0y) = \delta(4, y)$).

The probabilistic automaton in Figure 3 has *CII* with confidence 5/9:

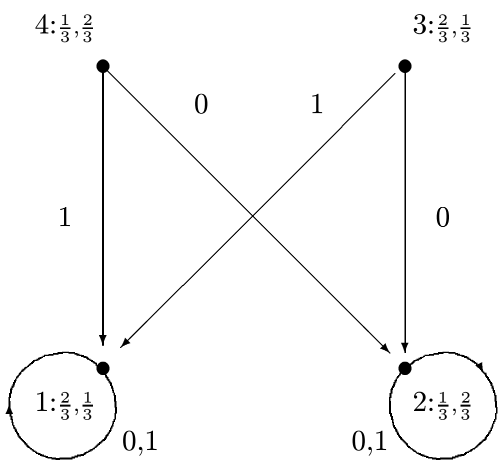


Figure 3.

Indeed, the following pairs of states are distinguishable with confidence 5/9 by every experiment: $(1, 2)$, $(1, 4)$, $(2, 3)$, $(3, 4)$. Accordingly, with confidence 5/9, 1 is distinguishable from the other states by 0, 2 is distinguishable by 1, 3 is distinguishable by 0, and 4 is distinguishable by 1, so the probabilistic automaton has property **B** with confidence 5/9. It does not have property **C** with confidence 5/9 because:

- any experiment which starts with 1, i.e. $1x$, $x \in \Sigma^*$, does not distinguish with confidence 5/9 between 1 and 3;
- any experiment which starts with 0, i.e. $0y$, $y \in \Sigma^*$, does not distinguish with confidence 5/9 between 2 and 4.

6 More About Mermin's Probabilistic Automata

We now argue that *with confidence 1/2, for every transition function δ, the corresponding Mermin probabilistic automaton has* **C**. Indeed,

- for every $i \neq j$, the probability that $f(ii) \neq f(jj) = 1/2$,
- for every $j \neq k$, the probability that $f(ii) \neq f(jk) = 7/8$,

- for every $i \neq j, k \neq l$, $ij \neq kl$, the probability that $f(ij) \neq f(kl) = 11/16$.

As all probabilities calculated above are greater than 1/2, it follows that with confidence 1/2, every Mermin automaton has **C**.

7 Decidability Questions

With techniques similar to those in Calude, Calude, Svozil, Yu [7] one can show that properties **A**, **B**, **C**, *CI*, *CII* are decidable for every probabilistic automaton. Complementarity properties *CI, CII* cannot appear for probabilistic automata with less than four states.

One way to check these properties, with some approximation, is by "simulating" a probabilistic automaton with a deterministic automaton (with some level of confidence). Let $\mathcal{A} = (Q, \Sigma, O, \delta, (a_{p,o})_{p \in Q, o \in O})$ be a probabilistic automaton, and $\alpha \in [1/2, 1]$ a confidence level. We construct a deterministic automaton $\mathcal{A}' = (Q, \Sigma, O', \delta, f')$, where $f : Q \to O'$ is the output function satisfying the following constraints: for every pair of distinct states p, q, if the probability that $f(p) \neq f(q)$ is greater or equal to α, then $f'(p) \neq f'(q)$; otherwise, $f'(p) = f'(q)$.

Note that the above construction cannot be carried on in all cases. For example, consider the probabilistic automaton $\mathcal{A} = (Q, \Sigma, O, \delta, (a_{p,o})_{p \in Q, o \in O})$ where $Q = \{p, q, r\}$, $O = \{G, R\}$, and the output probabilities are $a_{p,G} = 1, a_{p,R} = 0, a_{q,G} = 0, a_{q,R} = 1, a_{r,G} = a_{r,R} = 1/4$ (Σ, δ are arbitrary). It is easy to see that no deterministic automaton $\mathcal{A}'$ simulates $\mathcal{A}$, for every $\alpha \geq 1/2$. Reason: $f'(p) \neq f'(q)$, but $f'(p) = f'(q) = f'(r)$. However, this phenomenon cannot appear for all $\alpha \in [1/2, 1]$.

If the simulation is possible at the level of confidence α, then *every pair of distinct states $p, q \in Q$ are distinguishable by an experiment applied to $\mathcal{A}$ if and only if they are distinguishable by the same experiment applied to $\mathcal{A}'$*. Consequently, the probabilistic automaton $\mathcal{A}$ has **A** (**B**,**C**) if and only if $\mathcal{A}'$ has **A** (**B**, **C**, respectively).

For example, every Mermin probabilistic automaton is simulated with confidence $\alpha \leq 11/16 \approx 68.7\%$ by the automaton having the same components as every Mermin automaton and the output function $f'(11) = f'(22) = f'(33) = 0, f'(12) = f'(13) = f'(21) = f'(23) = f'(31) = f'(32) = 1$. Modifying the output function to $f'(11) = f'(22) = f'(33) = 0, f'(12) = 1, f'(13) = 2, f'(21) = 3, f'(23) = 4, f'(31) = 5, f'(32) = 6$ we get a simulation with confidence $\alpha \leq 7/8 \approx 87.5\%$. An analysis of properties **C,** *CI* and *CII* for these automata can be found in [5].[9]

If $\mathcal{A}$ has only two output states, say $\{G, R\}$, and $\alpha > 1/2$, then O' needs no more than two states. Indeed, O' needs more than two states if there are three distinct states $p, q, s \in Q$ such that

$$\text{the probability that } f(x) \neq f(y) \text{ is greater or equal to } \alpha, \tag{1}$$

[9] Due to the large number of transitions, i.e., $9^{18} \approx 150 \cdot 10^{15}$, an exhaustive search was computationally not feasible; instead, simulation techniques were used.

for all distinct $x, y \in \{p, q, s\}$. Assume that the probability that $f(x) = G$ is $a_{x,G}$, $x \in \{p, q, s\}$. Then, condition (1) can be written as

$$a_{x,G}(1 - a_{y,G}) + a_{y,G}(1 - a_{x,G}) \geq \alpha,$$

or

$$(a_{x,G} - \alpha)(\alpha - a_{y,G}) \geq 2\alpha - 1.$$

We arrive to a system of three inequalities which has no solution for $\alpha > 1/2$. The system has an infinity of solutions for $\alpha = 1/2$. For example, to simulate the probabilistic automaton $\mathcal{A} = (Q, \Sigma, O, \delta, (a_{p,o})_{p \in Q, o \in O})$ where $Q = \{1, 2, 3\}$, Σ, δ are arbitrary, $O = \{G, R\}$ and $f(1) = G$ with probability 1, and $f(2) = f(3) = G$ with probability 1/2, we need a deterministic automaton with three outputs.

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